

2022

M.Sc. (Bio-Informatics) First Semester

MBIN-8002: Mathematics

Time allowed: 3 Hours

Max. Marks: 60

**NOTE:** Attempt five questions in all, including Question No. 1 which is compulsory and selecting atleast one question from each Section.

x-x-x

**I (a)** How many terms are there in geometric progression

2, 4, 8, ..., 128.

**(b)** Simplify the surd  $\frac{\sqrt{2000}}{\sqrt{50}}$ .

**(c)** Find  $\vec{B} \times \vec{A}$  if  $\vec{A} = 2\vec{i} + \vec{j} + \vec{k}$ ,  $\vec{B} = -4\vec{i} + 3\vec{j} + \vec{k}$ .

**(d)** Find the sum of the first 50 terms of the sequence 1, 3, 5, 7, 9, ...

**(e)** Find the expansion of  $(2x - y)^4$ .

**(f)** For  $f(x) = x+1$  and  $g(x) = \sqrt{5x}$ , find  $g \circ f(1)$ .

**(g)** Determine the coefficient of  $x^3$  in the expansion of  $\frac{1}{(1+x)^2}$ .

**(h)** For  $z_1 = 6 - 2i$ ,  $z_2 = 3 + 4i$ . Find  $z_1 z_2$ .

(8 × 1.5)

### Section A

**II (a)** An arithmetic progression has 3 as its first term. Also the sum of the first 8 term. Also, the sum of the first 8 terms is twice the sum of the first 5 terms. Find the common difference. (6)

**(b)** From the 26 letters in English, 11 have vertical symmetry, 9 have horizontal symmetry and 4 have both types of symmetries. How many have neither? (6)

**III(a)** Solve each of the following equations for the complex number  $z$ .

(i)  $4 + 5i = z - (1 - i)$

(ii)  $(1 + 2i)z = 2 + 5i$

(3+3)

**(b)** Find the term independent of  $x$  in the expansion of  $(\frac{\sqrt{x}}{\sqrt{3}} + \frac{\sqrt{3}}{2x^2})^{10}$ . (6)

### Section B

**IV(a)** Evaluate  $\int \frac{-2x+4}{(x^2+1)(x-1)^2} dx$ . (6)

**(b)** Evaluate  $\lim_{x \rightarrow \infty} \frac{x^3+1}{3x^3-4x+5}$ . (6)

P.T.O.

(2)

V(a) (i) If  $y = (x+2)(x^2 + 1)$ . Find  $\frac{dy}{dx}$ .

(ii) If  $y = \log(x + \sqrt{x^2 + a^2})$ . Prove that  $\frac{dy}{dx} = \frac{1}{\sqrt{x^2 + a^2}}$ . (3+3)

(b) If  $y = a \cos mx + b \sin mx$ . Prove that  $\frac{d^2y}{dx^2} + m^2y = 0$ . (6)

### Section C

VI(a) If  $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 3 \\ -3 & -2 & 4 \end{bmatrix}$ . Find  $A^{-1}$ . (6)

(b) Determine the value of  $\lambda$  for which the system of equations

$$x + y + z = 1$$

$$x + 2y + 4z = \lambda$$

$$x + 4y + 10z = \lambda^2$$

possesses a solution and hence find a solution. (6)

VII(a) Find a unit vector which is coplanar with  $\vec{a}$  and  $\vec{b}$  but is perpendicular to  $\vec{c}$

$$\vec{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$$

$$\vec{b} = -\mathbf{i} + \mathbf{j} + \mathbf{k}$$

$$\vec{c} = \mathbf{i} + 2\mathbf{k}$$

(6)

(b) Construct a truth table to show that  $\overline{(X + YZ)} = \bar{X} \cdot (\bar{Y} + \bar{Z})$ . (6)

X-X-X