Exam.Code:0472
Sub. Code: 3702

2012

M.Sc. (Physics)

First Semester

PHY-8011: Mathematical Physics – I

Time allowed: 3 Hours Max. Marks: 60

***NOTE**: Attempt five questions in all, including Question No. 9 (Unit-V) which is compulsory and selecting one question each from Unit I - IV.*

x-x-x

UNIT-I

1. (a) Construct the function of complex variable $f(z) = u + iv$, if the real part $u = -r^3 sin3\theta$.

(b) Evaluate residue of the function

$$\frac{ze^{iz}}{z^4+a^4} ;\ a > 0$$

(c) Find the Laurent's series expansion of the function $(z) = \frac{1}{(z+1)(z+3)}$,

valid for $1 < |z| < 3$.

(4, 4, 4)

2. (a) Using Cauchy's residue theorem, prove that

$$\int_0^{\infty} \frac{x^{\alpha-1}}{1+x} dx = \frac{\pi}{sin\ \alpha\pi} \text{ if } 0 < \alpha < 1$$

(b) State and prove Laurent's series expansion for the singled valued analytic function of complex variable on two concentric circles C_1 and C_2 with centre at Z_0. **(6, 6)**

UNIT-II

3. (a) Give various definitions of Γ function and show that Weistrass's form leads to an important identity

$$\Gamma(z).\Gamma(1-z) = \frac{\pi}{sin\ \pi z}$$

(b) Prove that

$$\delta(f(x)) = \left|\frac{df(x)}{dx}\right|^{-1} .\delta(x - x_o)$$

where x_o is chosen so that $f(x_o) = 0$ with $\frac{df}{dx} \neq 0$; that is $f(x)$ has simple zero at x_o. **(6, 6)**

4. (a) Obtain the relation between beta and gamma function.

(b) Using the property of gamma function, prove that

$$\int_0^{\infty} x^{2s} e^{-ax^2} dx = \frac{\Gamma\left(s+\frac{1}{2}\right)}{2a^{s+\frac{1}{2}}}$$

(c) Evaluate the integral using beta function $\int_0^2 \frac{x^6}{(4-x^2)^{3/2}} dx$ **(4, 4, 4)**

P.T.O.

UNIT - III

5. (a) Solve the differential equation

$$x\frac{d^2y}{dx^2} + y = 0$$, by series solution method.

(b) Find the inverse of matrix

$$\begin{bmatrix} 2 & 4 & 6 \\ 4 & 2 & 3 \\ 3 & -3 & 1 \end{bmatrix}$$ **(6, 6)**

6. (a) Obtain the solution for two dimensional Laplace's differential equation in cylindrical co-ordinates and define circular harmonics.

(b) Find the nature of singularities associated with Laguerre's differential equation

$$xy'' + (1-x)y' + ay = 0$$

(c) Express the perturbed electrostatic potential of a conducting sphere placed in a uniform electric field in term of Legendre's polynomial. **(4, 4, 4)**

UNIT-IV

7. (a) Starting from generating function for Bessel's function $[J_n(x)]$ show that

$\cos x = J_0(x) - 2J_2(x) + 2J_4(x)$-......... and $\sin x = 2J_1(x) - 2J_3(x) + 2J_5(x)$-.........

(b) Prove the recurrence relations for Laguerre's $[L_n(x)]$ polynomials of n^{th} order

$$xL_n'(x) = nL_n(x) - nL_{n-1}(x)$$ **(6, 6)**

8. (a) Derive the expression of Rodrique's formula

$$H_x(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}\left(e^{-x^2}\right)$$ for Hermite's polynomial.

(b) Obtain the Orthogonality conditions for Legendre's polynomials. **(6, 6)**

UNIT-V

9. (a) Define zero of m^{th} order for the function of complex variables.

(b) Prove that $\vec{F} = (2xy + z^3)\hat{\imath} + x^2\hat{\jmath} + 3xz^2\hat{k}$ is an example conservative force field.

(c) What are the importances of dispersion relations in Physics? Quote the expressions for dispersion relations.

(d) Write down the generating functions for (i) Bessel's and (ii) Laguerre's function.

(e) Show that $x\delta(x) = 0$, where $\delta(x)$ is Dirac Delta function.

(f) Show that the value of $P_n(1) = 1$. **(2, 2, 2, 2, 2, 2)**