(i) **Printed Pages: 3** **Roll No. ....**

(ii) **Questions : 8** **Sub. Code : 0 0 4 4**

Exam. Code : 0 0 0 1

B.A./B.Sc. (General) 1st Semester

(1129)

MATHEMATICS

Paper—II

(Calculus—I)

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt ***five*** questions in all, selecting at least ***two*** questions from each of the Unit I and II.

UNIT—I

I. (a) Between any two distinct real numbers, there is always an irrational number, and therefore, infinitely many irrational numbers.

(b) If $|x - 5| < 1$, then prove that : $\dfrac{x^2-2x-1}{x-3} \in \left(\dfrac{17}{3}, 9\right)$.

3,3

II. (a) Prove that $\lim\limits_{x \to c} \dfrac{1}{x-c}$ does not exist.

(b) Find l.u.b and g.l.b., if exists, for the set :

$$\left\{\frac{2+x}{1-x} : x > 0, x \neq 1\right\}$$

3,3

III. (a) If a function f is continuous at $x = c$ and $f(c) \neq 0$, then prove that there exists a neighbourhood of c, where f(x) and f(c) has the same sign.

(b) Show that the function f defined by :

$$f(x) = \begin{cases} [x-3] + [3-x], & x \neq 3 \\ 0 & , x = 3 \end{cases}$$

is discontinuous at $x = 3$. 3,3

IV. (a) Evaluate :

$$\lim_{x \to 1} \frac{x^x - x}{x - 1 - \log x}$$

(b) Find the values of a, b and c, if

$$\lim_{x \to 0} \frac{(a + b\cos x)x - c\sin x}{x^5} = 1$$ 3,3

UNIT—II

V. (a) State and prove Cauchy's mean value theorem.

(b) Use mean value theorem to prove that :

$$\frac{\pi}{6} + \frac{2x-1}{\sqrt{3}} \leq \sin^{-1} x \leq \frac{\pi}{6} + \frac{2x-1}{2\sqrt{1-x^2}} \text{ for } \frac{1}{2} \leq x < 1.$$ 3,3

VI. (a) Differentiate w.r.t x

$$e^{\tanh^{-1}\left(\frac{2x}{1-x^2}\right)} + \sinh^{-1}(\operatorname{sech} x).$$

(b) Use Taylor's theorem to express :

$f(x) = 2 + x^2 - 3x^5 + 7x^6$ in powers of $(x - 1)$.

3,3

VII. (a) Show that $\coth^{-1}x = \frac{1}{2}\log\left(\frac{x+1}{x-1}\right)$ for $|x|>1$ and hence find its derivative.

(b) Use Maclaurin's theorem to prove

$$\sin^2 x = x^2 - \frac{x^4}{3} + \frac{2}{45}x^6 - \text{.........}$$ 3,3

VIII. (a) If $\sqrt{x}+\sqrt{y}=\sqrt{a}$, show that $\frac{d^2y}{dx^2}=\frac{1}{2a}$ at $x = a$.

(b) If $x = \cos\left(\frac{1}{m}\log y\right)$, prove that :

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0.$$ 3,3