(i) **Printed Pages: 3** **Roll No. ....**

(ii) **Questions : 8** **Sub. Code : 0 0 4 5**

Exam. Code : 0 0 0 1

B.A./B.Sc. (General) 1st Semester

(1129)

MATHEMATICS

Paper—III

(Trigonometry and Matrices)

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— (1) Attempt ***five*** questions in all by selecting at least ***two*** questions from each unit.

(2) All questions carry equal marks.

UNIT—I

1. (a) If $a = \cos\left(\frac{2\pi}{7}\right) + i\sin\left(\frac{2\pi}{7}\right)$, $b = a + a^2 + a^4$ and $c = a^3 + a^5 + a^6$, then show that b and c are roots of equation $x^2 + x + 2 = 0$. 3

(b) Find all value of $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{3/4}$ and show that continued product of all the value is 1. 3

2. (a) State and prove De-Moivre's theorem for integral index. 3

(b) Show that each primitive 12^{th} root of unity statisfies $x^4 - x^2 + 1 = 0$. 3

3. (a) If $i^{i^{i^{\cdots\infty}}} = A + iB$ and only principal value are considered, prove that :

(i) $\tan\left(\frac{\pi A}{2}\right) = \frac{B}{A}$

(ii) $A^2 + B^2 = e^{-\pi B}$. 3

(b) Prove that $\log\left(\frac{\sin(x+iy)}{\sin(x-iy)}\right) = 2i\tan^{-1}(\cot x \tanh y)$.

3

4. (a) Sum to n terms the series $\sin\theta + \frac{1}{3}\sin 2\theta + \frac{1}{3^2}\sin 3\theta + \ldots$

3

(b) Prove that $\lim_{x\to 0}\frac{1}{x^2}\log\left(\frac{\tan^{-1}x}{x}\right) = \frac{-1}{3}$. 3

UNIT—II

5. (a) Prove that every square matrix over C can be expressed uniquely as P + iQ, where P and Q are Hermitian matrices.

3

(b) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$. Find non-singular matrices P and Q such that PAQ is in normal form and hence determine rank of A. 3

6. (a) Solve completely the system of equation :

$$x + 2y + 2z - s + 3t = 0$$

$$x + 2y + 3z + s + t = 0$$

$$3x + 6y + 8z + s + 5t = 0$$ 3

(b) Show that rank of $\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ is less than 3 iff either $a + b + c = 0$ or $a = b = c$. 3

7. (a) Investigate for what value of γ, μ the simultaneous equations :

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$x + 2y + \gamma z = \mu$ have

(i) Unique solution

(ii) No solution

(iii) Infinite number of solutions. 3

(b) Prove that characteristic roots of Hermitian matrix are real. 3

8. (a) If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ then using Caley-Hamilton theorem express $A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2$ as linear polynomial in A. 3

(b) Diagonalize the following matrices, if possible :

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$ 3