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(ii) Questions : 8

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B.A./B.Sc. (General) 3rd Semester

(1129)

MATHEMATICS

Paper : (I. Advanced Calculus-I)

Time Allowed : Three Hours] [Maximum Marks : 30

Note :— Attempt **FIVE** questions in all selecting at least **TWO** questions each Section. All questions carry equal marks.

SECTION—A

1. (a) By using definition, prove that :

$$\lim_{(x,y)\to(1,2)} x^2 + 5y = 11.$$

(b) Show that the function defined by :

$$f(x, y) = \frac{x^3 - y^3}{x^2 + y^2} \quad (x, y) \neq (0, 0)$$

$$0 \quad (x, y) = (0, 0)$$

is continuous at (0, 0). 3,3

2. (a) If $V = r^m$ where $r=\sqrt{x^2+y^2+z^2}$, show that

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}=m(m+1)\,r^{m-2}.$$

(b) Discuss the differentiability of the function $f(x, y) = (xy)^{1/3}$ at (0, 0). 3,3

3. (a) Let $f(x,y)=\frac{xy(x-y)}{x+y}$ $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$ show that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$.

(b) If $\vec{r}=t^3\hat{i}+\left(2t^3-\frac{1}{5t^2}\right)\hat{j}$

show that $\vec{r}\times\frac{d\vec{r}}{dt}=\vec{k}$. 3,3

4. (a) In what direction from (3, 1, –2) is the directional derivative of $\phi = x^2y^2z^4$ maximum ? Find the magnitude of this maximum.

(b) Show that :

$$\nabla^2\left(\frac{x}{r^3}\right)=0.$$ 3,3

SECTION—B

5. (a) If $u = \sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, prove that :

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{-\sin u\cos 2u}{4\cos^3 u}.$$

(b) Expand $x^4 + x^2y^2 - y^4$ in the neighbourhood of the point (1, 1) upto the terms of second degree. 3,3

6. (a) If u, v, w be the roots of equation

$$(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0.$$

Prove that :

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = \frac{-2(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}.$$

(b) Show that the functions $u = x^2 + y^2 + z^2$, $v = xy - xz - yz$, $w = x + y - z$ are dependent and find a relation connecting them. 3,3

7. (a) Find the envelope of the circles passing through origin and whose centre lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

(b) Find the evolute of the parabola $y^2 = 4ax$. 3,3

8. (a) Prove that $u = x^2y^2 - 5x^2 - 8xy - 5y^2$ is maximum at $x = y = 0$.

(b) Prove that of all rectangular parallelopipeds of the same volume, the cube has the least surface. 3,3