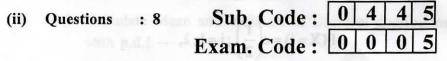


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Roll No.



B.A./B.Sc. (General) 5th Semester

(1129) (1129)

MATHEMATICS

Paper-III : Probability Theory

Time Allowed : Three Hours] [Maximum Marks : 30

Note :— Attempt FIVE questions in all, selecting at least TWO questions from each section.

SECTION-A

1. (a) Prove that for any n events A_1, A_2, \dots, A_n ,

we have
$$P\left(\bigcup_{i=1}^{n} A_{i}\right) \leq \sum_{i=1}^{n} P(A_{i})$$
.

(b) An urn contains 10 white and 3 black balls, while another urn contains 3 white and 5 black balls. Two balls are drawn from the first urn and put into the second urn and then a ball is drawn from the second urn. What is the probability that it is a white ball ? 3,3

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2. (a) Let X be a discrete random variable with

$$P(X=i) = c\left(\frac{1}{2}\right)^{i}; i = 1, 2,$$

Find (i) c, (ii) mean, variance and most probable value of the random variable X.

(b) The distribution function for a random variable X is :

$$F(x) = \begin{cases} 1 - e^{-2x}, & x \ge 0 \\ 0, & x < 0 \end{cases}$$

Find (i) the density function, (ii) the probability that X > 2, (iii) the probability that -3 < X ≤ 4. 3,3
3. (a) The p.d.f. of a random variable X is given by :

$$f(x) = \begin{cases} \frac{x+1}{2}, & -1 < x < 1\\ 0, & \text{elsewhere} \end{cases}$$

find the coefficient of skewness and hence describe the nature of the distribution.

- (b) Find the Moment generating function of Geometric distribution. Hence find its mean and variance. 3,3
- 4. (a) Show that in a Poisson distribution with unit mean,

mean deviation about mean is $\left(\frac{2}{e}\right)$ times the standard deviation.

(b) There are 5% defective items in a large bulk of items.
 Find the probability that a sample of 5 times will include not more than 2 defective items.
 3,3

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SECTION-B

5. (a) Calculate Mean and Variance of Uniform distribution with p.d.f.

$$f(x) = \begin{cases} \frac{1}{2h}, & 10 - h < x < 10 + h \\ 0, & \text{elsewhere.} \end{cases}$$

Also find its distribution function.

- (b) A random variable X has exponential distribution with parameter λ = 2. Find (i) P(X ≥ 5), (ii) S.D. and coefficient of variation.
- (a) Prove that Poisson distribution tends to normal distribution if its parameter m becomes very large.
 - (b) If X ~ N (75, 25), find the conditional probability that X is greater than 80 relative to the hypothesis that X is greater than 77.
 3,3
- 7. (a) Let X and Y have a joint p.d.f. as

$$f(x, y) = \begin{cases} 6x^2y, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find (i) P(X + Y < 1), (ii) P(X > Y), (iii) P(X < 1 | Y < 2).

(b) Show that coefficient of correlation is independent of scale and origin.
 3,3

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8. (a) The joint probability density function of bivariate random variables (X, Y) is given by :

$$f(x, y) = \begin{cases} 4xye^{-(x^2 + y^2)}, & x \ge 0, y \ge 0\\ 0, & \text{elsewhere.} \end{cases}$$

Find :

- (i) Whether X and Y are independent or not ?
- (ii) Conditional probability density function of X givenY = y.
 - (b) Let X and Y have a bivariate normal distribution with parameter

$$\mu_{\rm X} = 5, \ \mu_{\rm Y} = 10, \ \sigma_{\rm X}^2 = 1, \sigma_{\rm Y}^2 = 25 \ \text{and} \ \rho > 0.$$

If P(4 < Y < 16 | X = 5) = 0.954, then find r. 3,3

(b) If X > N (75, 35), find the conditional probability that
 X is greater than 80 relative to the hypothesis that X is greater than 77.

(a) Let X and Y have a joint p.d.f. as

$$f(x,y) = \begin{cases} 6x^2y, \ 0 < x < 1, 0 < y < 1\\ 0, \ clsewhere. \end{cases}$$

Find (i) P(X + Y < 1), (ii) P(X > Y), (iii) P(X < 1 | Y < 2). Show that coefficient of conclution is independent of scale and origin.