(i) Printed Pages: 4 Roll No. ....

(ii) Questions : 8

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B.A./B.Sc. (General) 5th Semester

(1129)

MATHEMATICS

Paper-III : Probability Theory

Time Allowed : Three Hours] [Maximum Marks : 30

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** questions from each section.

SECTION—A

1. (a) Prove that for any n events $A_1, A_2, \ldots\ldots A_n$,

$$\text{we have } P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i).$$

(b) An urn contains 10 white and 3 black balls, while another urn contains 3 white and 5 black balls. Two balls are drawn from the first urn and put into the second urn and then a ball is drawn from the second urn. What is the probability that it is a white ball ? 3,3

2. (a) Let X be a discrete random variable with

$$P(X=i)=c\left(\frac{1}{2}\right)^{i};i=1,2,.....$$

Find (i) c, (ii) mean, variance and most probable value of the random variable X.

(b) The distribution function for a random variable X is :

$$F(x)=\begin{cases}1-e^{-2x}, & x\geq 0\\ 0, & x<0\end{cases}$$

Find (i) the density function, (ii) the probability that $X > 2$, (iii) the probability that $-3 < X \leq 4$. 3,3

3. (a) The p.d.f. of a random variable X is given by :

$$f(x)=\begin{cases}\frac{x+1}{2}, & -1<x<1\\ 0, & \text{elsewhere}\end{cases}$$

find the coefficient of skewness and hence describe the nature of the distribution.

(b) Find the Moment generating function of Geometric distribution. Hence find its mean and variance. 3,3

4. (a) Show that in a Poisson distribution with unit mean, mean deviation about mean is $\left(\frac{2}{e}\right)$ times the standard deviation.

(b) There are 5% defective items in a large bulk of items. Find the probability that a sample of 5 times will include not more than 2 defective items. 3,3

SECTION—B

5. (a) Calculate Mean and Variance of Uniform distribution with p.d.f.

$$f(x)=\begin{cases}\frac{1}{2h}, & 10-h<x<10+h\\ 0, & \text{elsewhere.}\end{cases}$$

Also find its distribution function.

(b) A random variable X has exponential distribution with parameter $\lambda = 2$. Find (i) $P(X \geq 5)$, (ii) S.D. and coefficient of variation. 3,3

6. (a) Prove that Poisson distribution tends to normal distribution if its parameter m becomes very large.

(b) If $X \sim N(75, 25)$, find the conditional probability that X is greater than 80 relative to the hypothesis that X is greater than 77. 3,3

7. (a) Let X and Y have a joint p.d.f. as

$$f(x, y)=\begin{cases}6x^2y, & 0<x<1, 0<y<1\\ 0, & \text{elsewhere.}\end{cases}$$

Find (i) $P(X + Y < 1)$, (ii) $P(X > Y)$, (iii) $P(X < 1 \mid Y < 2)$.

(b) Show that coefficient of correlation is independent of scale and origin. 3,3

8. (a) The joint probability density function of bivariate random variables (X, Y) is given by :

$$f(x, y) = \begin{cases} 4xye^{-(x^2+y^2)}, & x \geq 0, y \geq 0 \\ 0 & , \text{elsewhere.} \end{cases}$$

Find :

(i) Whether X and Y are independent or not ?

(ii) Conditional probability density function of X given Y = y.

(b) Let X and Y have a bivariate normal distribution with parameter

$$\mu_X = 5, \mu_Y = 10, \sigma_X^2 = 1, \sigma_Y^2 = 25 \text{ and } \rho > 0.$$

If $P(4 < Y < 16 \mid X = 5) = 0.954$, then find r. 3,3