B.Sc. (Hons.) Biotechnology
First Semester
BIOT-Sem-I-III-T (a): Mathematics

Time allowed: 3 Hours **Max. Marks: 67**

***NOTE**: Attempt five questions in all, including Question No. 9 (Section-C) which is compulsory and selecting two questions each from Section A-B.*

x-x-x

Section A

1. (*a*) Find the modulus and principal argument of the complex number $-\sqrt{3}+i$. [4]

(*b*) Convert $\frac{1+7i}{(1-2i)^2}$ in the polar form. [4]

(*c*) If $z_1 = 2-i$, $z_2 = 1+i$ then find $|\frac{z_1+z_2+1}{z_1-z_2+1}|$. [5]

2. (*a*) Prove that $\sqrt{5}$ is an irrational number. [4]

(*b*) Find the value of $\lim_{x\to 3} \frac{x-3}{\sqrt{x-2}-\sqrt{4-x}}$. [4]

(*c*) Evalute $\lim_{x\to 1} \frac{sin(1-x)}{\sqrt{1-x^2}}$. [5]

3. (*a*) Find the union of the following pairs of sets A=$\{x : x$ is a natural number and $1 < x \leq 6\}$ B=$\{x : x$ is a natural number and $6 < x < 10\}$. [4]

(*b*) Let $f, g : R \to R$ be defined by $f(x) = x^2 + 1$ and $g(x) = x + 2$ then find fog and gof. [4]

(*c*) Draw the graph of the function $f(x) = x^2; x \in R$. [5]

4. (*a*) Find the domain of the function $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$. [4]

(*b*) Evaluate $\lim_{x\to 1}(\frac{2}{1-x^2} - \frac{1}{x-1})$. [4]

(*c*) Find all the points of discontinuity of the function defined by

$$f(x) = \begin{cases} x+2 & \text{if } x < 1 \\ 0 & \text{if } x = 1 \\ x-2 & \text{if } x > 1 \end{cases}$$

[5]

Section B

5. (*a*) Find $\frac{dy}{dx}$ if $ax + by^2 = \cos y$. [5]

(*b*) Differenciate the given function with respect to x, where $f(x) = (\log x)^{\cos x}$. [4]

(*c*) Find the local maximum and minimum values of the function f given by $f(x) = 2x^3 - 6x^2 + 6x + 5$. [4]

P.T.O.

6. (a) Evaluate $\int x \sin x dx$. [4]

(b) Using integration find the area of the region bounded by the triangle whose vertices are (-1,0), (1,3) and (3,2). [5]

(c) Find $\frac{dy}{dx}$ when $x = a(\theta - \cos 2\theta)$, $y = \sin\theta - \sin 2\theta$. [4]

7. (a) Solve the following Linear Programming Problem graphically: Maximise $Z = 5x + 3y$ subject to $3x + 5y \leq 15, 5x + 2y \leq 10, x \geq 0, y \geq 0$ [7]

(b) Consider the marks, out of 100, obtained by 39 students of a class in a test, given in the Table below. Draw a frequency polygon for the given data.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of students	5	10	4	6	7	3	2	2

[6]

8. (a) Calculate the mean, Variance and Standard Deviation for the following distribution. [7]

Classes	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

(b) From the prices of shares X and Y below, find out which is more stable in value: [6]

X	35	54	52	53	56	58	52	50	51	49
Y	108	107	105	105	106	107	104	103	104	101

Section C

9. (a) Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$ find $(A \cup B)' and (A')'$ [2]

(b) Evaluate $\int \frac{dx}{\sqrt{5x^2 - 2x}}$. [2]

(c) Find the mediam for the following data: 23, 12, 18, 14, 75, 43, 32, 18. [2]

(d) Form a differential equation representing the given family of curves by eleminating arbitrary constants a and b, $y = ae^{3x} + be^{-2x}$. [3]

(e) Find the general solution of the differential equation $\frac{dy}{dx} = \frac{x+1}{2-y}$. [2]

(f) Find the dgree and order of the differential equation $\frac{d^3y}{d^3x} - \sqrt{\frac{dy}{dx}} + 3$. [2]

(g) Evaluate $\int x \log 2x dx$. [2]

x-x-x