(i) Printed Pages: 3 Roll No. .....

(ii) Questions : 8 Sub. Code : 0043

Exam. Code : 0001

R-19

B.A./B.Sc. (General) 1st Semester

(01)

1128

MATHEMATICS

Paper-I : Plane Geometry

Time Allowed : Three Hours] [Maximum Marks : 30

Note :— Attempt **five** questions in all by selecting at least **two** questions from each section.

SECTION—A

I. (i) Transform the equation $3x^2 + 2xy + 3y^2 + 18x + 22y + 50 = 0$ to the form $Ax^2 + By^2 = C$ by the suitable transformation of axes. 3

(ii) Show that if $ax^2 + 2hxy + by^2 = 1$ and $a'x^2 + 2h'xy + b'y^2 = 1$ represent the same conic and axes are rectangular, then $(a - b)^2 + 4h^2 = (a' - b')^2 + 4h'^2$. 3

II. (i) Prove that the straight lines joining the origin to the points of intersection of straight lines $2x - 3y + 4 = 0$ with the curve $x^2 + 4xy + 2y^2 + 12x + 4y = 0$ are at right angles. 3

(ii) Find the equation of straight lines bisecting the angle between straight lines $ax^2 + 2hxy + by^2 = 0$. 3

III. (i) Prove that the two circles :

$x^2 + y^2 + 2ax + c = 0$, $x^2 + y^2 + 2by + c = 0$ where $a^2, b^2 > c$ touches if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$. 3

(ii) Find the equation of the circle through the points of intersection of the circles $x^2 + y^2 + 6x + 4y - 12 = 0$ and $x^2 + y^2 - 4x - 6y - 12 = 0$ and cutting the circle $x^2 + y^2 - 2x + 3 = 0$ orthogonally. 3

IV. (i) Find the radical axis and limiting points of co-axial system determined by circles $x^2 + y^2 - 6x - 6y + 4 = 0$ and $x^2 + y^2 - 2x - 4y + 3 = 0$. 3

(ii) Find the locus of mid-points of the chords of the circle $x^2 + y^2 = 16$ which touches the circle $(x - 4)^2 + (y - 3)^2 = 36$. 3

SECTION—B

V. (i) Prove that the locus of middle points of the normal chords of the parabola $y^2 = 4ax$ is $\frac{y^2}{2a} + \frac{4a^3}{y^2} = x - 2a$. 3

(ii) Prove that the locus of the middle points of parallel chords of parabola is a straight line parallel to axis. 3

VI. (i) Show that the equation of director circle of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $x^2 + y^2 = a^2 + b^2$. 3

(ii) Find the length of the semi-diameter conjugate to the diameter $y = 3x$ of the ellipse $9x^2 + 4y^2 = 36$. 3

VII. (i) Show that the locus of the poles of the normal chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is the curve $\frac{a^6}{x^2} - \frac{b^2}{y^2} = (a^2 + b^2)^2$. 3

(ii) Find the joint equation of asymptotes to the hyperbola $3x^2 - 5xy - 2y^2 + 5x + 11y - 8 = 0$. Also find the equation of conjugate hyperbola. 3

VIII. (i) Show that the tangents at the extremities of a focal chord of a parabola intersect each other perpendicular on the directrix. 3

(ii) Identify the curve $x^2 - 4xy + 4y^2 - 32x + 4y + 16 = 0$ and find its vertex and focus. 3