

B.A./B.Sc. (General) 1st Semester

1128

MATHEMATICS

Paper : II Calculus-I

Time Allowed : Three Hours]

[Maximum Marks : 30

Note :— Attempt five questions, selecting at least two from each section.

SECTION—A

- I. (a) Prove that between any two distinct real numbers, there exist infinitely many irrational numbers.

- (b) Solve :

$$|x - 2| > -5.$$

4,2

- II. (a) Prove that the set :

$$S = \left\{ \frac{1-x}{1+x} : x \leq 0, x \neq -1 \right\}$$

is unbounded.

- (b) If a set is bounded, then does its
- $\ell.u.b.$
- and
- $g.\ell.b.$
- belong to set ?

4,2

- III. (a) Does least upper bound property hold for rational numbers ? Justify.

- (b) If
- $f(x) = x \left[\frac{1}{x} \right]$
- , does

$$\lim_{x \rightarrow \frac{1}{3}} f(x) \text{ exist ?}$$

4,2

IV. (a) Evaluate :

$$\lim_{x \rightarrow \infty} \frac{\sin^{-1}\left(\frac{1}{x}\right)}{\tan\left(\frac{1}{x}\right)}.$$

(b) Find a, b, c so that :

$$\lim_{x \rightarrow \infty} \frac{axe^x - b \sin x + cxe^{-x}}{x^3} = 2. \quad 2,4$$

SECTION—B

V. (a) Expand $\tan^{-1}x$ in powers of x .

(b) Prove that :

$$|\sin x - \sin y| \leq |x - y| \quad \forall x, y \in \mathbb{R}. \quad 3,3$$

VI. State and prove Rolle's theorem. Interpret geometrically. 6

VII. (a) If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$. Find

$$\frac{d^2y}{dx^2} \text{ at } t = \frac{\pi}{4}.$$

(b) State and prove Leibnitz's theorem. 3,3

VIII. (a) If $f(x) = e^{\tanh^{-1}\left(\frac{2x}{1+x^2}\right)} + (\cosh x)^x$

then find $f'(x)$. 3,3

(b) Prove that :

$$\frac{d}{dx} (\operatorname{sech}^{-1}x) = \frac{-1}{x\sqrt{1-x^2}}; 0 < x < 1. \quad 3,3$$