(i) Printed Pages: 3 Roll No. ....

(ii) Questions : 8 Sub. Code : | 0 | 0 | 4 | 5 |

Exam. Code : | 0 | 0 | 0 | 1 |

B.A./B.Sc. (General) 1st Semester

1128

MATHEMATICS

Paper-III : Trigonometry and Matrices

Time Allowed : Three Hours] [Maximum Marks : 30

Note :— Attempt **five** questions in all by selecting at least **two** questions from each unit.

UNIT—I

1. (a) If $a = \text{cis } \alpha$, $b = \text{cis } \beta$, $c = \text{cis } y$ and $a + b + c = 0$. Then prove that :

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0.$$

(b) If α and β are the roots of $x^2 - 2x + 4 = 0$, prove that :

$$\alpha^n+\beta^n=2^{n+1}\cos\frac{n\pi}{3}.$$ 3,3

2. (a) Solve $x^7 = 1$ and prove that the sum of the n^{th} powers of the roots is 7 or zero according as n is or not multiple of 7.

(b) Prove that :

$$\cos^7\theta=\frac{1}{2^6}[\cos 7\theta + 7\cos 5\theta + 21\cos 3\theta + 35\cos\theta].$$

3,3

3. (a) If $\sin(\theta + i\phi) = \tan\alpha + i\sec\alpha$, show that :

$$\cos 2\theta \cosh 2\phi = 3.$$

(b) If $\cos(\theta + i\phi) = r(\mathrm{Cos}\,\alpha + i\sin\alpha)$, then prove that :

$$\phi = \frac{1}{2}\log\frac{\sin(\theta-\alpha)}{\sin(\theta+\alpha)}.$$ 3,3

4. (a) For $\alpha, \beta \in C$, $\beta \neq 2n\pi$, $n \in z$, show that :

$$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+\alpha\beta) + \dots\dots\dots + \cos(2\alpha(n-1)\beta)$$

$$= \frac{\cos\left(\alpha + \frac{n-1}{2}\beta\right)\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}.$$

(b) Prove that :

$$1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \dots\dots = \frac{\pi}{2\sqrt{2}}.$$ 3,3

UNIT—II

5. (a) Show that every Hermitian Matrix A can be uniquely expressed as $P + iQ$, where P and Q are real symmetric and real skew symmetric matrices respectively. Also show that $A^{\theta}A$ is real iff $PQ = -QP$.

(b) Check for the linear dependence of the following system of vectors : $u = (1, -1, 1)$, $v = (2, 1, 1)$, $w = (3, 0, 2)$. If dependent, find the relation between them. 3,3

6. (a) Find the rank of the matrix $\begin{bmatrix} 9 & 0 & 2 & 3 \\ 0 & 1 & 5 & 6 \\ 4 & 5 & 3 & 0 \end{bmatrix}$ by reducing it to normal form.

(b) Express the following matrix as the sum of a Hermitian and Skew Hermitian matrix :

$$\begin{bmatrix} 2-i & 3 & 1+i \\ -5 & 0 & -6i \\ 7 & i & -3+2i \end{bmatrix}.$$ 3,3

7. (a) Find the value of k so that the equations :

$$x - 2y + z = 0$$
$$3x - y + 2z = 0$$
$$y + kz = 0 \text{ have}$$

(i) a unique solution, (ii) infinitely many solutions. Also find solutions for these values of k.

(b) Find values of λ and μ for which the system of equations :

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu \text{ has}$$

(i) no solution, (ii) a unique solution, (iii) an infinite number of solutions. 3,3

8. (a) State and prove Cayley-Hamilton theorem.

(b) Check whether the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & 4 & 9 \end{bmatrix}$ is diagonalizable or not. 3,3