(i) **Printed Pages : 3** **Roll No. ....**

(ii) **Questions : 9**

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B.A./B.Sc. (General) 3rd Semester

1128

MATHEMATICS

Paper : I (Advanced Calculus–I)

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt any **five** questions in all selecting at least **two** questions each from Unit-**I** and Unit-**II** each question carry equal marks.

UNIT—I

1. (a) Let $f: R^2 \to R$ be defined by

$f(x,y) = x \sin\frac{1}{y} + y \sin\frac{1}{x},\ x \neq 0,\ y \neq 0$. Prove that

$\lim\limits_{(x,y)\to(0,0)} f(x,y) = 0$.

(b) Discuss the continuity of the following function at (0,0) :

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

2. (a) If $z(x + y) = x^2 + y^2$, then show that

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right).$$

(b) Show that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$, where

$$f(x,y) = \begin{cases} x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\frac{x}{y} & \text{if } xy \neq 0 \\ 0 & \text{if } xy = 0 \end{cases}$$

3. (a) If $u = e^x \sin y$, $x = \log t$, $y = t^2$, then by partial differentiation find $\frac{du}{dt}$. Also verify by direct calculations.

(b) Show that $f(x, y) = \cos(x + y)$ is differentiable at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$.

4. (a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.

(b) Define irrotational vector. Find constants a, b and c for which $\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational.

UNIT—II

5. (a) If $z = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that :

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = -\frac{1}{2}\cot z.$$

(b) Use Taylor's theorem to expand $x^2 y + 3y - 2$ in powers of $x - 1$ and $y + 2$.

6. (a) If $u^3 + v + w = x + y^2 + z^2$, $u + v^3 + w = x^2 + y + z^2$, $u + v + w^3 = x^2 + y^2 + z$, prove that :

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = \frac{1-4(yz+zx+xy)+16xyz}{2-3(u^2+v^2+w^2)+27u^2v^2w^2}.$$

(b) Show that the functions $u = \frac{x}{y-z}$, $v = \frac{y}{z-x}$ and $w = \frac{z}{x-y}$ are not independent of each other and also find the relation between them.

7. (a) Find the envelope of a system of concentric and co-exial ellipses of constant area.

(b) Show that the evolute of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is $(x+y)^{\frac{2}{3}} + (x-y)^{\frac{2}{3}} = 2a^{\frac{2}{3}}$.

8. (a) Find all the points of maximum and minimum of the function :

$f(x,y) = x^3 + y^3 - 63(x+y) + 12xy$.

Also discuss the saddle points of the function.

(b) Find the maximum and minimum value of $x^2 + y^2$ subject to the condition $3x^2 + 4xy + 6y^2 = 140$.