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B.A./B.Sc. (General) 3rd Semester

1128

MATHEMATICS

Paper—III : Statics

Time Allowed : Three Hours] [Maximum Marks : 30

Note :— Attempt **five** questions, selecting at least **two** from each section. All questions carry equal marks.

SECTION—I

I. (a) The resultant of two forces P and Q acting at a point is equal to $\sqrt{3}$ Q and makes an angle of 30° with the direction of P. Show that P = Q or P = 2Q.

(b) Two forces P and Q acting on a particle are inclined at an angle θ. If the sum of their resolved parts along a certain direction is X and that along a direction perpendicular to it is Y, show that

$$\theta = \cos^{-1}\left(\frac{X^2 + Y^2 - P^2 - Q^2}{2PQ}\right).$$ 3,3

II. (a) The ends of an inelastic and weightless string 17 cm long are attached to two points 13 cm apart in same horizontal line. Find the maximum weight that can be suspended from the string at a distance 5 cm from one end, it is known that string cannot bear a tension greater than 1 kg.

(b) If forces of magnitude P, Q and R act at a point parallel to the sides BC, CA and AB respectively of a ΔABC then show that the magnitude of their resultant force is $\sqrt{P^2 + Q^2 + R^2 - 2QR\cos A - 2RP\cos B - 2PQ\cos C}$.

3,3

III. (a) Find a point O within a triangle ABC, so that forces represented by OA, OB and OC in magnitude and direction and acting at O may be in equilibrium.

(b) A body is sustained on a smooth inclined plane by two forces each equal to half of the weight of the body in magnitude, one acting horizontally and other acting along the plane. Find the inclination of plane. 3,3

IV. (a) A rod AB of length a + b and weight W has its center of gravity at a distance a from A. It rests on two parallel knife-edges at a distance c apart in the same horizontal line so that equal portions of rod are projected beyond each knife-edge. Show that the pressures on knife-edges are $\frac{b-a+c}{2c}W$ and $\frac{a-b+c}{2c}W$.

(b) The resultant of two like parallel forces P and Q passes through a point C. When P is increased by R and Q by S, the resultant still passes through C and also when Q, R replace P, Q respectively. Show that

$$S = R - \frac{(Q-R)^2}{P-Q}.$$ 3,3

SECTION—II

V. (a) A man on the ground is pulling a vertical tree with a given force by means of an inelastic rope of length p metre and of negligible weight. At what point of the tree, the one end of the rope must be tied, so that the man pulling it from the other end have greatest tendency to pull it over ?

(b) Forces of magnitudes P, Q and R act along the sides BC, CA and AB respectively of a triangle ABC. If their resultant passes through the circumcenter of triangle ABC then show that :

$$P \cos A + Q \cos B + R \cos C = 0.$$ 3,3

VI. (a) P, Q, R are points taken on the sides BC, CA, AB of a triangle ABC, dividing the respective sides in the ratio $1 + \lambda : 1 - \lambda$ in the same sense. Prove that the forces represented completely by AP, BQ, CR form a couple of moment $2\lambda\Delta$ in magnitude, where Δ is the area of triangle ABC.

(b) Forces of magnitude $5\sqrt{3}$ kg, 5 kg and 9 kg act along the sides BC, CA and AB of an isosceles triangle ABC with AB = AC and angle A = 120°. Find their resultant in magnitude and direction. Also find the point where the line of action of the resultant meets BC. 3,3

VII. (a) Two uniform rods AB and BC having same weight per unit length are rigidly joined at B so that $\angle ABC = 90°$. The rods are suspended freely from A. The lengths of the rods are a and b respectively. If in position of equilibrium, AB makes an angle θ with the vertical then show that :

$$\tan\theta = \frac{b^2}{a^2 + 2ab}.$$

(b) A heavy rod of weight W is hung from a point by two equal strings, whose other ends are attached to the extremities of the rod. A weight ω is suspended half way between the mid point and one end of the rod. If T_1 and T_2 are the tensions in two strings then show that

$$\frac{T_1}{T_2} = \frac{2W + 3\omega}{2W + \omega}.$$ 3,3

VIII. (a) A body of weight W can be just sustained on a rough inclined plane by a force P acting along the plane or by a horizontal force Q, show that the coefficient of friction is $P\left[\frac{1}{W^2} + \frac{1}{Q^2} - \frac{1}{P^2}\right]^{1/2}$.

(b) A uniform rod rests in limiting equilibrium within a rough hollow sphere. If the rod subtends a right angle at the centre of the sphere then show that the inclination of the rod with the horizontal is twice the angle of friction. 3,3