(i) Printed Pages : 3 Roll No. ....

(ii) Questions : 8 Sub. Code : 0 4 4 3

Exam. Code : 0 0 0 5

B.A./B.Sc. (General) 5th Semester

1128

MATHEMATICS

Paper–I : Analysis—I

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :– Attempt **FIVE** questions in all, selecting at least **two** questions from each Section. All questions carry equal marks.

SECTION—A

1. (a) Show that set of irrational numbers is uncountable.

(b) By considering the integral $\int_n^{n+1} \frac{1}{x}\, dx, n > 0$ prove that

$$\frac{1}{n+1} \le \log\left(1+\frac{1}{n}\right) \le \frac{1}{n}.$$

2. (a) If f is integrable on [a, b] and c is a real number then cf is integrable on [a, b]. Moreover $\int_a^b cf\, dx = c\int_a^b f\, dx$.

(b) Proceeding from the definition, compute $\int_1^2 \frac{1}{x}\, dx$.

3. (a) State and prove fundamental theorem of Integral Calculus.

(b) Prove that $\int_{-1}^{\infty} \frac{x+1}{(x+2)^6}\, dx = \frac{1}{20}$.

4. (a) Prove that $B(m, n) = \frac{\Gamma m \, \Gamma n}{\Gamma m+n}, m > 0, n > 0$.

(b) Prove that $\int_0^\infty x^n e^{-a^2x^2} dx = \frac{1}{2a^{n+1}} \frac{\Gamma(n+1)}{2}$, where $n + 1 > 0$.

Hence evaluate $\int_{-\infty}^{\infty} e^{-a^2x^2} dx$.

SECTION—B

5. (a) Discuss the convergence of $\int_0^\infty \frac{x \log x}{(1+x^2)^2} dx$.

(b) Show that improper integral $\int_0^1 \frac{dx}{\sqrt{x - x^2}}$ is convergent and its value is π.

6. (a) If $\phi(x)$ is bounded and monotonic in $[a, \infty)$ and tends to 0 as $x \to \infty$ and $\int_a^t f(x)\, dx$ is bounded for all $t \geq a$, then $\int_a^\infty f(x)\, \phi(x)\, dx$ is convergent at ∞.

(b) Use Abel's test to show that $\int_0^\infty e^{-ax} \frac{\sin x}{x} dx, \; a \geq 0$ is convergent.

7. (a) Show that $\int_0^{\frac{\pi}{2}} x^m \operatorname{cosec}^n dx$ exist if and only if $n < m + 1$.

(b) Show that $\int_0^\infty \frac{ae^{-ce^{ax}}}{1-e^{-ax}} - \frac{be^{-ce^{bx}}}{1-e^{-bx}} dx = e^{-c} \log\frac{b}{a}$.

where a, b, c > 0

8. (a) Evaluate $\int_0^\infty \frac{e^{-ax} \sin bx}{x} dx$, where $a \geq 0$. Hence deduce that

$$\int_0^\infty \frac{\sin bx}{x} dx = \frac{\pi}{2}.$$

(b) Show that for $x^2 \leq 1$.

$$\int_0^\infty \log(1-a^2\cos^2 x)dx = \pi \log\left(1+\sqrt{1-a^2}\right) - \pi \log 2.$$