1128
B.Sc. (Hons.) Biotechnology
First Semester
BIOT-Sem-I-III-T (a): Mathematics

Time allowed: 3 Hours

Max. Marks: 67

NOTE: Attempt *five* questions in all, including Question No. I (Section-A) which is compulsory and selecting two questions each from Section B-C.

x-x-x

Section-A

1. (a) Prove that $\sqrt{3}$ is an irrational number.

   (b) Simplify : $\dfrac{(64)^{2/3} \times (81)^{3/4}}{(27)^{2/3}}$

   (c) Express $\dfrac{5-\sqrt{3}}{2+\sqrt{3}}$ in the form $a + b\sqrt{3}$, where a, b are integers.

   (5,4,4)

2. (a) Convert $\dfrac{1+7i}{(2-i)^2}$ in the Polar form.

   (b) If $x + iy = \dfrac{(a+i)^2}{2a^2+1}$, prove that $x^2 + y^2 = \dfrac{(a^2+1)^2}{(2a^2+1)^2}$

   (c) If $\left(\dfrac{1}{1-2i} + \dfrac{3}{1+i}\right)\left(\dfrac{3+4i}{2-4i}\right) = a + ib$, find a and b.

   (5,4,4)

3. (a) A college awarded 38 medals in football, 15 in basketball and 20 in Cricket. If these medals went to a total of 58 students and only three students got medals in all the three sports, how many students received medals in exactly two of the three sports?

   (b) Let $A = \{1,2,3,4,5\}$. Define a relation R on A defined by :
   $R = \{(x,y): y = 2x - 1\}$
   (i) Write R in roster form.
   (ii) Depict this relation using an arrow diagram.
   (iii) Write down the domain, codomain and range of R.

   (c) Draw the graph of the modulus function. Also, write its domain and range.

   (5,4,4)

P.T.O.

4. (a) *Evaluate*: $\lim_{x \to 0} \frac{(\sin 7x + 6x)}{(7x + \sin 6x)}$

   (b) *Suppose* $f(x) = \begin{cases} 5 & , \quad x \leq 2 \\ ax + b & , \quad 2 < x < 10 \\ 21 & , \quad x \geq 10 \end{cases}$ *and if* $\lim_{x \to 2} f(x)$ *and* $\lim_{x \to 10} f(x)$
   both exists, find the values of a and b.

   (c) *Evaluate* : $\lim_{x \to 1} \left(\frac{2}{1-x^2} + \frac{1}{x-1} \right)$

   (5,4,4)

Section-B

5. (a) *Find* $\frac{dy}{dx}$, *if* $y^x + x^y = a^b$

   (b) *A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cm^3 of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15cm.*

   (c) *Find local maximum and local minimum values of the function f given by* $f(x) = x^3 - 6x^2 + 9x + 15$

   (5,4,4)

6. (a) *Evaluat* : $\int e^{2x} \sin x \, dx$

   (b) *Find the area bounded by the curve* $x^2 = 4y$ *and the straight line* $x = 4y - 2.$

   (c) *Solve* : $(x + y - 5) \frac{dx}{dy} = 1$

   (5,4,4)

7. (a) Solve the following Linear Programming Problem graphically:
   Maximize $z = 1000x + 600y$
   Subject to : $x + y \leq 200$
   $\qquad\qquad\quad x \geq 20$
   $\qquad\qquad\quad y \geq 4x$
   $\qquad\qquad\quad x, y \geq 0$

   (b) *In a study of diabetic patients, the following data was obtained:*

Age(in years)	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of patients	3	8	30	36	27	15	6

   Draw the histogram and a frequency curve for the above data.

   (7,6)

(a) *Calculate the mean, variance and standard deviation for the following frequecy distribution:*

Classes	0-30	30-60	60-90	90-120	120-150	150-180	180-210
Frequency	2	3	5	10	3	5	2

8. (b) *Calculate median, first quartile and 85th percentile for the following data:*

Classes	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	75	250	350	192	68	35	24	6

(7,6)

Section – C
(Compulsory Questions)

9. (a) *Let* $A = \{1,2,3,4,5\}$, $B = \{1,3,5\}$, *Find* $A - B$ *and* $B - A$. (2)

(b) *Simplify* : $\dfrac{(5-4i)(4+5i)}{(4+3i)}$ (2)

(c) *Differentiate* $e^{\sin^2 x}$ *w.r.t* $\sin x$. (2)

(d) *Evaluate* $\int \dfrac{\sqrt{\tan x}}{\sin x \cos x} dx$ (2)

(e) *Show that the function* $f(x) = \log \sin x$ *is strictly increasing on* $\left(0, \dfrac{\pi}{2}\right)$. (2)

(f) *Form a differential equation of the family of curves*
$\dfrac{x}{a} + \dfrac{y}{b} = 1$, *by eliminating the arbitary constants a and b.* (2)

(g) *Find the median, lower quartile and upper quartile of the following* (3)
numbers: 5 , 22 , 6 , 3 , 7 , 33 , 9

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