(i) Printed Pages: 3 Roll No. .....

(ii) Questions : 8 Sub. Code : 0145

Exam. Code : 0002

B.A./B.Sc. (General) 2nd Semester

1059

MATHEMATICS

Paper : I Solid Geometry

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt **five** questions, selecting at least **two** questions from each section. All questions carry equal marks.

SECTION—A

1. (a) Shift the origin to a suitable point so that the equation :

$$2x^2 - 2y^2 + z^2 - 4x + 8y + 2z - 5 = 0$$

is transformed into an equation in which the first degree terms are absent.

(b) Transform the equation :

$$13x^2 + 13y^2 + 10z^2 + 8xy - 4yz - 4zx - 144 = 0$$

when the axes are rotated to the axes having directions cosines :

$$\left\langle \frac{-1}{3}, \frac{2}{3}, \frac{2}{3} \right\rangle, \left\langle \frac{2}{3}, \frac{-1}{3}, \frac{2}{3} \right\rangle, \left\langle \frac{2}{3}, \frac{2}{3}, \frac{-1}{3} \right\rangle$$

2. (a) Obtain the equation of a sphere having its centre on the line $5y + 2z = 0 = 2x - 3y$ and passing through the points $(0, -2, -4)$ and $(2, -1, -1)$.

(b) Prove that the plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius unity; and find the equation of the sphere which has this circle for one of the great circles.

3. (a) Show that the line $\frac{x-7}{2} = \frac{y-4}{7} = \frac{z-13}{10}$ touches the sphere $x^2 + y^2 + z^2 - 6x + 2y - 4z + 5 = 0$. Find the coordinates of P, the point of contact.

(b) Find the limiting points of the coaxial system defined by the spheres $x^2 + y^2 + z^2 + 3x - 3y + 6 = 0$ and $x^2 + y^2 + z^2 - 6y - 6z + 6 = 0$.

4. (a) Find the equation of the right circular cylinder of radius z whose axis is the line $\frac{x-1}{2} = \frac{y}{1} = \frac{z-3}{2}$.

(b) Find the equation of the cylinder whose generators are parallel to z-axis and which passes through the curve of intersection of the surfaces represented by $x^2 + y^2 + 2z^2 = 12$ and $x + y + z = 1$.

SECTION—B

5. (a) Find equation of the cone with vertex at the origin and which passes through the curve given by :

$$x^2 + y^2 + z^2 + x - 2y + 3z = 4 \text{ and}$$
$$x^2 + y^2 + z^2 + 2x - 3y + 4z = 5.$$

(b) Find the equation of the right circular cone whose vertex is at the origin, whose axis is line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and which has a vertical angle of 60°.

6. (a) Find the equation of the quadric cone which passes through the three coordinates axes and the three mutually perpendicular lines $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$, $\frac{x}{1} = \frac{y}{-1} = \frac{z}{-1}$, $\frac{x}{5} = \frac{y}{4} = \frac{z}{1}$.

(b) Find the equation of the cone whose vertex is (2, –3, 1) and whose guiding curve is $4x^2 + y^2 = 1$, $z = 0$.

7. (a) Show that the plane $x = 0$ cuts the enveloping cone of the sphere $x^2 + y^2 + z^2 = 11$ which has its vertex at (1, 4, 2) in a rectangular hyperbola.

(b) Show that $2y^2 - 8yz - 4zx - 8xy + 6x - 4y - 2z + 5 = 0$ represents a cone; find the coordinates of its vertex.

8. (a) Identify the following surface :

$$16z^2 - 4x^2 - 8z + 8x - 3 = 0.$$

(b) Show that the curve :

$$3x^2 - y^2 + z^2 + 18x - 16y - 8z - 3 = 0$$

represents hyperboloid of two sheets.