(i) **Printed Pages: 3** **Roll No. .....**

(ii) **Questions : 8** **Sub. Code :** | 0 | 1 | 4 | 6 |

Exam. Code : | 0 | 0 | 0 | 2 |

B.A./B.Sc. (General) 2nd Semester

1059

MATHEMATICS

Paper : II Calculus-II

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt **FIVE** questions in all selecting at least **TWO** questions from each unit. Each question carries **6** marks.

UNIT—I

1. (a) Find the intervals in which the curve

$$y = \frac{x^2 + 1}{x^2 - 1}$$

is concave upwards and concave downwards.

(b) Find the points of inflexion of the curve

$$y = (\sin 4x + \cos 4x)\, e^{4x},\ 0 < x < \frac{\pi}{2}.$$ 3,3

2. (a) Find the position and nature of double points on the curve

$$(x - 2)^2 = y(y - 1)^2.$$

(b) Find the equation of the cubic, which has the same asymptotes as the curve

$$x^3 - 6x^2y + 11xy^2 - 6y^3 + 4x + 5y + 7 = 0$$ and

which passes through the points (0, 0), (–2, 0) and (0, –2). 3,3

3. (a) Trace the curve $y = x + \frac{1}{x}$.

(b) Prove that the curvature of a circle is constant and is equal to the reciprocal of the radius. 3,3

4. (a) Find the co-ordinates of the centre of curvature at any point (x, y) of the parabola $y^2 = 4ax$. Also find its evolute.

(b) If c_x, c_y be the lengths of the chords of curvature parallel to the co-ordinate axes at any point of the curve

$y = c\cosh\frac{x}{c}$, then prove that $4c^2\left(c_x^2 + c_y^2\right) = c_y^4$. 3,3

UNIT—II

5. (a) Evaluate $\int \frac{2\sinh x + 3\cosh x}{\cosh x + 2\sinh x}\,dx$.

(b) If $I_{m,n} = \int_0^{\pi/2} \cos^m x \sin nx\,dx$, then show that

$$I_{m,n} = \frac{1}{m+n} + \frac{m}{m+n} I_{m-1,n-1} \qquad \begin{bmatrix} m,n \in I^+ \\ m+n \neq 0 \end{bmatrix}$$

Hence evaluate $I_{5,3}$. 3,3

6. (a) Use Trapezoidal Rule to evaluate

$$\int_0^{\pi/4} \sin 4x \, dx$$ by dividing the interval $[0, \pi/4]$

into four equal subintervals.

(b) Evaluate $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \left(\sin \frac{\pi r}{2n} \right)^{2k}$. 3,3

7. (a) Find the area above the x-axis included between the curves $y^2 = 2ax - x^2$ and $y^2 = ax$.

(b) Find the length of the loop of the curve

$$9ay^2 = x(x - 3a)^2, \; a > 0.$$ 3,3

8. (a) Find the volume generated by revolving the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ about its major axis.

(b) Find the surface area of the solid obtained by revolving the arc of the curve $y = \sin x$ from $x = 0$ to $x = \Pi$ about x-axis. 3,3

(i) Printed Pages : 2 Roll No. .....

(ii) Questions : 8 Sub. Code : 0 1 4 7

Exam. Code : 0 0 0 2

B.A./B.Sc. (General) 2nd Semester

1059

MATHEMATICS

Paper–III Theory of Equations

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :– (1) Attempt **five** questions in all by selecting at least **two** questions from each unit.

(2) **All** questions carry equal marks.

UNIT–I

1. (a) Find a real polynomial f(x) of least degree having roots $-2, 1 + \ell$ and satisfying $f(3) = 15$. 3

   (b) Solve the equation $x^6 - 4x^5 - 11x^4 + 40x^3 + 11x^2 - 4x - 1 = 0$, given $\sqrt{2} + \sqrt{3}$ is root of equation. 3

2. (a) Solve the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$, given one root is negative of the other. 3

   (b) Solve $3x^4 - 40x^3 + 130x^2 - 120x + 27 = 0$, roots being in G.P. 3

3. (a) Let $f(x) = a_0 + a_1x + a_2x^2 + \ldots\ldots\ldots\ldots + a_nx^n$ be a real polynomial of degree n and a_0 is non zero.

Let r and s be the number of variations of sign in f(x) and f(–x) respectively. Show that n-r-s is even. 3

(b) Show that the equation $2x^7 + 3x^4 + 3x + k = 0$ has at least four non real roots for all values of k. 3

4. (a) Solve the equation $3x^3 - 22x^2 + 48x - 32 = 0$, given that roots are in H.P. 3

(b) Find the equation whose roots are squared difference of the roots of equation $x^3 + 6x^2 + 9x + 4 = 0$. Hence show that the given equation has double roots. 3

UNIT–II

5. (a) Prove that $\sqrt{5} - \sqrt{2}$ is an irrational number. 3

(b) Solve by Newton's method of divisors

$x^5 - 29x^4 - 31x^3 + 31x^2 - 32x + 60 = 0$. 3

6. (a) Solve the cubic $x^3 + 6x^2 + 9x + 4 = 0$ by Carden's method. 3

(b) Show that the parabola $y = x^2$ meets the hyperbola $xy + 8x + 4y + 3 = 0$ in a single point. 3

7. (a) Find the roots of equation $x^3 - 3x + 1 = 0$ by rigonometric Method. 3

(b) Use Newton's Method of approximation to find the positive root of $x^3 - x^2 - 3 = 0$ correct to four decimal places.3

8. (a) Solve the equation $x^4 - 8x^2 - 24x + 7 = 0$ by Descarte's method. 3

(b) Solve by Ferrari's method $x^4 - 4x^3 + 4x^2 - 4x + 3 = 0$. 3