(i) Printed Pages: 4 Roll No. ....

(ii) Questions : 8 **Sub. Code : 0 3 4 1**

Exam. Code : 0 0 0 4

B.A./B.Sc. (General) 4th Semester

1059

MATHEMATICS

Paper-I (Advanced Calculus-II)

Time Allowed : Three Hours] [Maximum Marks : 30

Note :— Attempt **five** questions in all, selecting at least **two** questions from each unit. All questions carry equal marks.

UNIT—I

1. (a) Prove that the sequence $\{a_n\}$, $a_n = \dfrac{3n-1}{4n+5}$ is :

   (i) monotonically increasing

   (ii) bounded above

   (iii) converges to $\dfrac{3}{4}$. 3

   (b) Prove that the sequence $\{a_n\}$ where :

$$a_n = \frac{1}{\sqrt{2n^2+1}} + \frac{1}{\sqrt{2n^2+2}} + \dots\dots\dots + \frac{1}{\sqrt{2n^2+n}}$$

   converges to $\dfrac{1}{\sqrt{2}}$. 3

2. (a) If s_1 and s_2 are positive and $s_{n+1} = \frac{1}{2}(s_n + s_{n-1})$ prove that the sequences $s_1, s_3, s_5, \ldots\ldots ; s_2, s_4, s_6, \ldots\ldots$ are the one increasing and the other decreasing and show that their common limit is $\frac{1}{3}(s_1 + 2s_2)$. 3

(b) If $x_n > 0 \ \forall\ n$ and $\lim_{n\to\infty} x_n = l$, then show that :

$$\lim_{n\to\infty} \sqrt[n]{x_1 x_2 x_3 \ldots\ldots x_n} = l.$$ 3

3. (a) State and prove Cauchy's general principle of convergence. 3

(b) Using the concept of subsequences, show that $\{(-1)^n\}$ is not convergent. 3

4. (a) Using the concept of sequential continuity, show that the function f defined by :

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

is continuous only at $x = 0$. 3

(b) Let $f : (0, \infty) \to \mathbb{R}$ be a function defined by $f(x) = \frac{1}{x}$. Prove that f is uniformly continuous on $[a, \infty)$, $a > 0$. 3

UNIT—II

5. (a) Show that the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 1$ and diverges for $0 < p < 1$. 3

(b) Discuss the convergence or divergence of $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^2+1}}\, x^n$. 3

6. (a) Discuss the convergence or divergence of the series

$$1+\frac{\alpha\beta}{1.\gamma}x+\frac{\alpha(\alpha+1)\beta(\beta+1)}{1.2.\gamma(\gamma+1)}x^2+\ldots\ldots\infty, x>0.$$ 3

(b) Show that the following series :

$$\frac{1}{\log 2}-\frac{1}{\log 3}+\frac{1}{\log 4}-\frac{1}{\log 5}+\ldots\ldots\ldots$$

is conditionally convergent. 3

7. (a) Examine the convergence or divergence of :

$$\sum_{n=1}^{\infty} e^{\sqrt{n}} r^n, \; r > 0.$$ 3

(b) Discuss the convergence or divergence of the series

$$x^2+\frac{2^2}{3.4}x^4+\frac{2^2.4^2}{3.4.5.6}x^6+\frac{2^2.4^2.6^2}{3.4.5.6.7.8}x^8+\ldots\ldots\ldots$$ 3

8. (a) What rearrangement of the series :

$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\ldots\ldots\ldots$ will reduce the sum to $\frac{1}{2}\log 2$? 3

(b) Prove that the series $\sum_{n=1}^{\infty} r^{\log n}$, $r > 0$ is convergent if $r < \frac{1}{e}$ and divergent if $r \geq \frac{1}{e}$. 3