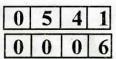
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(ii) Questions : 8

Sub. Code : 0 Exam. Code : 0



B.A./B.Sc. (General) 6th Semester

1059

MATHEMATICS

Paper-I : Analysis-II

Time Allowed : Three Hours

[Maximum Marks : 30

Note :— Attempt five questions in all, selecting two questions from each section. All questions carry equal marks.

SECTION-A

1. (a) Consider the region $A = \{(x, y) : 1 \le x \le 2, 3 \le y \le 4\}$. Let $f : A \rightarrow \mathbb{R}$ be defined by :

 $f(x, y) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational.} \end{cases}$

Show that f is not integrable over the region A.

(b) Evaluate :

 $\iint_A x^2 dxdy \text{ where A is the region enclosed by}$

the four parabolas $y^2 = ax$, $y^2 = bx$, $x^2 = cy$, $x^2 = dy$ where a, b, c, d are real numbers with b > a > 0, d > c > 0. 3+3=6

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[Turn over

- 2. (a) Prove that the volume enclosed by the cylinders $x^{2} + y^{2} = 2ax$, $z^{2} = 2ax$ is $\frac{128}{15}a^{3}$ units.
 - (b) Change the order of integration of $\int_{0}^{2a} \int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} f(x, y) dy dx$

Hence evaluate for f(x, y) = 1. 3+3=6

- 3. (a) State and prove Stoke's Theorem for vector point functions.
 - (b) If $\vec{F} = (2x^2 + y^2)\hat{i} + (3y 4x)\hat{j}$. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ around the triangle ABC whose vertices are A(0, 0), B(2, 0) and C(2, 1). 3+3=6
- 4. (a) Verify Divergence Theorem for $\vec{f} = 4x\hat{i} 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by the cylinder $x^2 + y^2 = 9$, z = 0, z = 4.
 - (b) Apply Green's Theorem in plane to evaluate $\oint_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$ where C is the boundary

of the surface enclosed by the X axis and the semi circle $y = \sqrt{1 - x^2}$. 3+3=6

SECTION-B

5. (a) Show that the series $\sum [\log (n+1)]^{-x} \cos nx$ is uniformly convergent in $[\theta_1, \theta_2]$ where $0 \le \theta_1, \theta_2 \le \pi$.

- (b) Show that the sequence $\left\{\frac{1}{x+n}\right\}$ is uniformly convergent in any interval [0, b] where b > 0. 3+3=6
- 6. (a) Let {f_n} be a sequence of real valued functions on E which converges uniformly to f on E. If each f_n is continuous on E then prove that f is also continuous on E.

(b) Show that the series
$$\sum_{n=1}^{\infty} \left[\frac{nx}{1+n^2x^2} - \frac{(n-1)x}{1+(n-1)^2x^2} \right] \operatorname{can}$$

be integrated term by term on [0, 1] although it is not uniformly convergent on [0, 1]. 3+3=6

7. (a) Find the Fourier series for f(x) on $(-\pi, \pi)$ when :

$$f(x) = \begin{cases} \pi + x & \text{if } -\pi < x < 0 \\ \pi - x & \text{if } 0 < x < \pi. \end{cases}$$

(b) Obtain the Fourier series in the interval $\begin{bmatrix} -\frac{1}{2}, \frac{1}{2} \end{bmatrix}$ of the function f(x) given by :

$$f(x) = \begin{cases} x - [x] - \frac{1}{2} & \text{if } x \text{ is not an integer} \\ 0 & \text{if } x \text{ is an integer} \end{cases}$$

where [x] is the greatest integer $\leq x$. 3+3=6

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8. Find the sum for $|x| \le 1$ of the power series :

$$x + \sum_{n=1}^{\infty} (-1)^n \frac{1 \cdot 3 \cdot 5 \dots \cdot (2n-1)}{2 \cdot 4 \cdot 6 \dots \cdot (2n)} \cdot \frac{x^{2n+1}}{2n+1}$$

and deduce that :

$$1 + \frac{1.3}{2.4} \cdot \frac{1}{5} + \frac{1.3.5}{2.4.6} \cdot \frac{1}{9} + \dots = \frac{\pi}{4} + \frac{1}{2} \log(1 + \sqrt{2}).$$

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