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B.A./B.Sc. (General) 6th Semester

1059

MATHEMATICS

Paper-I : Analysis-II

Time Allowed : Three Hours] [Maximum Marks : 30

Note :— Attempt **five** questions in all, selecting **two** questions from each section. All questions carry equal marks.

SECTION—A

1. (a) Consider the region $A = \{(x, y) : 1 \leq x \leq 2, 3 \leq y \leq 4\}$. Let $f : A \to \mathbb{R}$ be defined by :

$$f(x, y) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational.} \end{cases}$$

Show that f is not integrable over the region A.

(b) Evaluate :

$\iint\limits_A x^2 dxdy$ where A is the region enclosed by the four parabolas $y^2 = ax$, $y^2 = bx$, $x^2 = cy$, $x^2 = dy$ where a, b, c, d are real numbers with $b > a > 0$, $d > c > 0$. 3+3=6

2. (a) Prove that the volume enclosed by the cylinders $x^2 + y^2 = 2ax$, $z^2 = 2ax$ is $\frac{128}{15}a^3$ units.

(b) Change the order of integration of $\int_0^{2a}\int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} f(x, y)dydx$.

Hence evaluate for $f(x, y) = 1$. 3+3=6

3. (a) State and prove Stoke's Theorem for vector point functions.

(b) If $\vec{F} = (2x^2 + y^2)\hat{i} + (3y - 4x)\hat{j}$. Evaluate $\int_C \vec{F}.\vec{dr}$ around the triangle ABC whose vertices are A(0, 0), B(2, 0) and C(2, 1). 3+3=6

4. (a) Verify Divergence Theorem for $\vec{f} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by the cylinder $x^2 + y^2 = 9$, $z = 0$, $z = 4$.

(b) Apply Green's Theorem in plane to evaluate $\oint_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$ where C is the boundary of the surface enclosed by the X axis and the semi circle $y = \sqrt{1 - x^2}$. 3+3=6

SECTION—B

5. (a) Show that the series $\sum [\log(n+1)]^{-x} \cos nx$ is uniformly convergent in $[\theta_1, \theta_2]$ where $0 \le \theta_1, \theta_2 \le \pi$.

(b) Show that the sequence $\left\{\frac{1}{x+n}\right\}$ is uniformly convergent in any interval [0, b] where b > 0. 3+3=6

6. (a) Let $\{f_n\}$ be a sequence of real valued functions on E which converges uniformly to f on E. If each f_n is continuous on E then prove that f is also continuous on E.

(b) Show that the series $\sum_{n=1}^{\infty}\left[\frac{nx}{1+n^2x^2}-\frac{(n-1)x}{1+(n-1)^2x^2}\right]$ can be integrated term by term on [0, 1] although it is not uniformly convergent on [0, 1]. 3+3=6

7. (a) Find the Fourier series for f(x) on $(-\pi, \pi)$ when :

$$f(x)=\begin{cases}\pi+x & \text{if } -\pi<x<0\\ \pi-x & \text{if } 0<x<\pi.\end{cases}$$

(b) Obtain the Fourier series in the interval $\left[-\frac{1}{2},\frac{1}{2}\right]$ of the function f(x) given by :

$$f(x)=\begin{cases}x-[x]-\frac{1}{2} & \text{if } x \text{ is not an integer}\\ 0 & \text{if } x \text{ is an integer}\end{cases}$$

where [x] is the greatest integer $\leq$ x. 3+3=6

8. Find the sum for $|x| \le 1$ of the power series :

$$x + \sum_{n=1}^{\infty} (-1)^n \frac{1.3.5......(2n-1)}{2.4.6.......(2n)} \cdot \frac{x^{2n+1}}{2n+1}$$

and deduce that :

$$1 + \frac{1.3}{2.4} \cdot \frac{1}{5} + \frac{1.3.5}{2.4.6} \cdot \frac{1}{9} + ... = \frac{\pi}{4} + \frac{1}{2} \log(1+\sqrt{2}).$$

4+2=6