(i) **Printed Pages: 3** **Roll No. ....**

(ii) **Questions : 8** **Sub. Code : 0 5 4 2**

Exam. Code : 0 0 0 6

B.A./B.Sc. (General) 6th Semester

1059

MATHEMATICS

Paper-II (Linear Algebra)

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt any **five** questions in all, selecting at least **two** questions from each unit. All questions carry equal marks.

UNIT—I

1. (a) Define vector space. Give an example.

   (b) Discuss whether or not R^2 is a subspace of R^3. 4,2

2. (a) Let V be vector space of $n \times n$ matrices over the field of real numbers and W_1 and W_2 be subspaces of symmetric and skew-symmetric matrices of order n respectively. Show that :

   $$V = W_1 \oplus W_2.$$

   (b) Prove that the set of vectors $\{v_1, v_2, ..., v_n\}$ form a L.D. set if at least one off the vectors is a zero vector. 4,2

3. (a) Prove that there exists a basis for each finitely generated vector space.

   (b) Show that $B = \{(1, 1, 1), (1, -1, 1), (0, 1, 1)\}$ is a basis of R^3. 4,2

4. (a) Find a basis and dimension of the subspace W of R^4 generated by the vectors (1, –2, 5, –3), (2, 3, 1, –4), (3, 8, –3, –5). Also extend these to a basis of R^4.

(b) State Rank-Nullity theorem. Verify for $T : R^3 \to R^3$ be defined by $T(x, y, z) = (x + 2y, y - z, x + 2z)$.

3,3

UNIT—II

5. (a) Let $V = R^3$ and let $T : V \to V$ be the linear transformation defined by $T(x, y, z) = (2x, 4y, 5z)$. Find matrix of T with respect to the basis $\left(\frac{2}{3}, 0, 0\right)$, $\left(0, \frac{1}{2}, 0\right)$ and $\left(0, 0, \frac{1}{4}\right)$ of V.

(b) Let V(F) be n-dimensional vector space and $B = \{v_1, v_2, ..., v_n\}$ be basis of V(F). If T be a linear operator on V such that $[T; B] = [\alpha_{ij}]_{n \times n}$ for $\alpha_{ij} \in F$. Prove that T is invertible iff $[T]_B$ is invertible and $[T^{-1}; B] = [T; B]^{-1}$.

3,3

6. (a) Find all the eigen values and basis for each eigen space of linear operator $T : R^3 \to R^3$ defined by $T(x, y, z) = (3x + y + 4z, 2y + 6z, 5z)$.

(b) Prove that the characteristic vectors corresponding to distinct characteristic roots of a matrix are linearly independent.

3,3

7. (a) Prove that characteristic roots of a Skew-Hermitian matrix are either purely imaginary or zero.

(b) Show that eigen values of a triangular matrix are just the diagonal entries of the matrix. 3,3

8. (a) Prove that the characteristic and minimal polynomial of an operator of a matrix have same irreducible factors.

(b) Find the characteristic polynomial for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Find A^{-1} using Caylay Hamilton theorem. 3,3