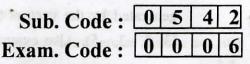
i) Printed Pages: 3 Roll No.

(ii) Questions : 8



B.A./B.Sc. (General) 6th Semester

1059

MATHEMATICS

Paper-II (Linear Algebra)

Time Allowed : Three Hours] [Maximum Marks : 30

Note :- Attempt any five questions in all, selecting at least two questions from each unit. All questions carry equal marks.

UNIT-I

- 1. (a) Define vector space. Give an example.
 - Discuss whether or not R^2 is a subspace of R^3 . 4.2 (b)
- 2. (a) Let V be vector space of $n \times n$ matrices over the field of real numbers and W, and W, be subspaces of symmetric and skew-symmetric matrices of order n respectively. Show that :

$$V = W_1 \oplus W_2$$
.

- (b) Prove that the set of vectors $\{v_1, v_2, \dots, v_n\}$ form a L.D. set if at least one off the vectors is a zero vector.
 - 4.2
- 3. (a) Prove that there exists a basis for each finitely generated vector space.
 - (b) Show that $B = \{(1, 1, 1), (1, -1, 1), (0, 1, 1)\}$ is a basis of R³. 4,2

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- 4. (a) Find a basis and dimension of the subspace W of R⁴ generated by the vectors (1, -2, 5, -3), (2, 3, 1, -4), (3, 8, -3, -5). Also extend these to a basis of R⁴.
 - (b) State Rank-Nullity theorem. Verify for $T : R^3 \rightarrow R^3$ be defined by T(x, y, z) = (x + 2y, y - z, x + 2z).

3,3

UNIT-II

5. (a) Let $V = R^3$ and let $T: V \rightarrow V$ be the linear transformation defined by T(x, y, z) = (2x, 4y, 5z). Find matrix

of T with respect to the basis $\left(\frac{2}{3}, 0, 0\right), \left(0, \frac{1}{2}, 0\right)$ and

- (b) Let V(F) be n-dimensional vector space and B = {v₁, v₂,, v_n} be basis of V(F). If T be a linear operator on V such that [T; B] = [α_{ij}]_{n×n} for α_{ij} ∈ F. Prove that T is invertible iff [T]_B is invertible and [T⁻¹; B] = [T; B]⁻¹.
- 6. (a) Find all the eigen values and basis for each eigen space of linear operator T : R³ → R³ defined by T(x, y, z) = (3x + y + 4z, 2y + 6z, 5z).
 - (b) Prove that the characteristic vectors corresponding to distinct characteristic roots of a matrix are linearly independent. 3,3

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- (a) Prove that characteristic roots of a Skew-Hermitian matrix are either purely imaginary or zero.
 - (b) Show that eigen values of a triangular matrix are just the diagonal entries of the matrix. 3,3
- (a) Prove that the characteristic and minimal polynomial of an operator of a matrix have same irreducible factors.
 - (b) Find the characteristic polynomial for the matrix

 $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$ Find A⁻¹ using Caylay Hamilton theorem. 3,3

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