(i) Printed Pages : 4] Roll No. .....

(ii) Questions : 8] **Sub. Code : 0044**

Exam. Code : 0001

B.A./B.Sc. (General) 1st Semester Examination

1127

MATHEMATICS

(Calculus-I)

Paper : II

Time : 3 Hours] **[Max. Marks : 30**

Note :– (i) Attempt *five* questions, selecting at least *two* questions from each Unit.

(ii) Each question will carry 6 marks.

Unit–I

1. (a) Solve the inequation :

$$\frac{2}{x-2} < \frac{x+2}{x-2} < 2.$$

(b) State and prove Archimedian property. Using the property prove that the set of natural numbers N is not bounded above. (3,3)

2. (a) Show that $\lim_{x \to 0} \sin \frac{1}{x}$ does not exists.

(b) Evaluate :

$$\lim_{x \to 1/2} \frac{1}{x} \left[\frac{1}{x} \right], \text{ if exists.} \qquad (3,3)$$

3. (a) Use intermediate value theorem to show that equation $\sin x - x + 1 = 0$ has a real root.

(b) Evaluate :

$$\lim_{x \to 0} \frac{x - \sin x}{\tan^3 x}. \qquad (3,3)$$

4. (a) Evaluate :

$$\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right).$$

(b) Discuss the continuity of

$$f(x) = \begin{cases} \dfrac{|x| + x}{3}, & x \leq 3 \\ \dfrac{2|x-3|}{x-3}, & x > 3 \end{cases} \text{ over R.} \qquad (3,3)$$

Unit–II

5. (a) Differentiate $y = x^{\sinh x} + x^{\cosh x}$ w.r.t. x.

(b) Let f be a real valued function defined in $[a, b]$ such that (i) f is continuous in $[a, b]$ (ii) f is differentiable in (a, b) (iii) $f(a) = f(b)$, then there exists at least one CE(a, b) such that $f'(c) = 0$. (2,4)

6. (a) Prove that $\tanh^{-1} x = \frac{1}{2}\log\left(\frac{x+1}{1-x}\right)$, $-1 < x < 1$, and then find its derivative.

(b) Use Cauchy's mean value theorem to evaluate

$$\lim_{x \to 1} \frac{\cos \pi x}{\frac{2}{\log 1}}.$$ (3,3)

7. (a) Use mean value theorem to prove :

$$\frac{x}{1+x} < \log(1+x) < x \text{ for } x > -1, x \neq 0.$$

(b) Use Taylor's theorem to express the polynomial

$2x^3 + 7x^2 + x - 6$ in powers of $(x - 2)$. (3,3)

8. (a) State and prove leibnitz's Theorem.

(b) If $y = \frac{\log x}{x}$, prove that

$$y_n = \frac{(-1)^n \lfloor n}{x^{n+1}} \left[\log x - 1 \frac{-1}{2} \frac{-1}{3} \ldots\ldots\ldots\ldots \frac{-1}{n} \right]. \quad (3,3)$$