(i) Printed Pages : 4]　　　　Roll No. ....

(ii) Questions : 8]　　　　**Sub. Code : 0 2 4 1**

Exam. Code : 0 0 0 3

B.A./B.Sc. (General) 3rd Semester Examination

1127

MATHEMATICS
(Advanced Calculus-I)
Paper : I

Time : 3 Hours] **[Max. Marks : 30**

Note :– Attempt *five* questions in all, selecting at least *two* questions from each Section. All questions carry equal marks.

Section–I

1. (a) Show that the function :

$$f(x, y) = \frac{x^2y^2}{x^2y^2 + (x-y)^2},$$

the two repeated limits at (0, 0) exist and are equal, but the simultaneous limit does not exist.

(b) Show that the function $f(x, y) = |x| + |y|$ is continuous at the origin.

2. (a) If $u = \frac{1}{\sqrt{1-2xy+y^2}}$, show that :

$$\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial u}{\partial x}\right]+\frac{\partial}{\partial y}\left(y^2\frac{\partial u}{\partial y}\right)=0$$

(b) If Z is function of x and y prove that if $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, then $\frac{\partial z}{\partial u}-\frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$.

3. (a) Show that $f(x, y) = \cos x + \cos y$ is differentiable at every point of R^2.

(b) For the function $\oint x^2y^3z^4$, find the directional derivative of ϕ at (2, 3, 1) in the direction making equal angles with x, y and z-axis.

4. (a) If $\vec{f} = x^2yz\,\hat{i} - 2xz^3\,\hat{j} + xz^2\,\hat{k}$ and $\vec{g} = 2z\,\hat{i} + y\,\hat{j} - x^2\,\hat{k}$, then find the value $\frac{\partial^2}{\partial x \partial y}(\vec{f}\times\vec{g})$ at point (2, 0, –3).

(b) Find constants a, b, c so that $\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational.

Section–II

5\. (a) If H = $f(x, y, z)$ is a homogeneous function of x, y and z of degree n then $x\frac{\partial H}{\partial x} + y\frac{\partial H}{\partial y} + z\frac{\partial H}{\partial z} = n\text{H}$.

(b) State Taylor's theorem for the function of two variables and use this to expand for $f(x, y) = e^{xy}$ at (1, 1) upto third term.

6\. (a) If $u^3 + v + w = x + y^2 + z^2$,

$$u + v^3 + w = x^2 + y + z^2,$$

$$u + v + w^3 = x^2 + y^2 + z,$$

prove that :

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{1 - 4(xy + yz + zx) + 16xyz}{2 - 3(u^2 + v^2 + w^2) + 27u^2v^2w^2}$$

(b) Show that $u = x + y + z$,

$$v = yz + zx + xy,$$

$$w = x^3 + y^3 + z^3 - 3xyz$$

are not independent of one another also find the relation between them.

7. (a) Find the envelope of the family of lines :

$$\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2,$$

θ being the parameter.

(b) Find the centre of curvature of the rectangular hyperbola $xy = a^2$ and deduce the equation of its evolute.

8. (a) Find the maximum and minimum values of the function :

$$f(x, y) = \sin x + \sin y + \sin (x + y)$$

(b) Let $f : \mathrm{R}^3 \rightarrow \mathrm{R}$ be defined by $f(x, y, z) = xyz$. Determine x, y, z for maximum of f subject to condition $xy + 2yz + 2zx = 108$.