(i) Printed Pages : 4] Roll No. .....

(ii) Questions : 8] **Sub. Code : 0243**

Exam. Code : 0003

B.A./B.Sc. (General) 3rd Semester Examination

1127

MATHEMATICS

(Statics)

Paper : III

Time : 3 Hours] **[Max. Marks : 30**

Note :– Attempt *five* questions in all, selecting at least *two* questions from each Unit. All questions will carry 6 marks.

Unit–I

1. (a) Prove that two non-zero forces acting on a particle are in equilibrium if they are of equal magnitude and opposite directions.

   (b) The greatest and least resultants of two forces can have are of magnitude P and Q respectively. Show that when they act at angle θ their resultant is of magnitude

$$\sqrt{P^2 \cos^2 \frac{\theta}{2} + Q^2 \sin^2 \frac{\theta}{2}}.$$

2. (a) If P and Q are two components of a given force F and its line of action divides the angle between them in the ratio 1 : 2, then show that $Q(F + Q) = P^2$.

   (b) A, B and C are three points on a circle, forces inversely proportional to AB and BC act along AB and BC respectively. Show that their resultant act along the tangent to the circle at B.

3. (a) State and prove Lami's theorem.

   (b) A body of mass 26 kg is suspended by two strings 5 cm and 12 cm long, their other ends being fastened to the extremities of a rod 13 cm long. If the rod be so held that the body hangs immediately below the middle point. Find the tensions in the strings.

4. (a) Two like parallel forces P and Q act at two points of a body. If Q be changed to $\frac{P^2}{Q}$, show that the line of action of the resultant is same as it would be if the forces are simply interchanged.

   (b) A body is sustained on a smooth inclined plane by two forces each equal to half of the weight in magnitude, one acting horizontally and the other acting along the plane. Find the inclination of the plane to the horizon.

Unit–II

5. (a) Forces of magnitude 5, 12 and 13N act resp. along the sides BC, CA and AB of a ΔABC. If $\angle B = \frac{\pi}{2}$, $\angle C = \frac{\pi}{3}$, find the moment of each of the forces about the opposite vertax.

   (b) Like parallel forces P, Q and R act at the angular points A, B and C of a triangle ABC. Prove that the perpendicular distance of their centre from the side BC is $\frac{P}{P+Q+R}\left(\frac{2\Delta}{a}\right)$, where Δ is the area of triangle ABC and BC $= a$.

6. (a) Forces of magnitude P, 2P, –P and 2P act along the sides AB, BC, CD and DA of a square ABCD and $\sqrt{2}P$ acts along each of the diagonals BD and CA, show that the forces are equivalent to a couple of moment $2aP$, where 'a' is side of the square.

   (b) A uniform rod of length $2a$ rests in equilibrium against a smooth vertical wall and upon a peg at a distance b from the wall. Show that the rod is inclined to the horizontal at an angle $\cos^{-1}\left(\frac{b}{a}\right)^{1/3}$.

7. (a) Show that two coplanar couples of equal and opposite moments are in equilibrium.

   (b) A uniform ladder of weight w rests with one end against a smooth vertical wall and other end resting on a smooth floor. If the inclination of the ladder to the horizon is 60°, find the horizontal force that must be applied at the lower end to prevent the ladder from slipping.

8. (a) Let a body of weight w be placed on a rough plane inclined to the horizontal at an angle α, which is greater than the angle of friction λ. If a force P be applied to the body acting at an angle θ with the inclined plane, prove that the body will be in equilibrium for :

$$w\frac{\sin(\alpha-\lambda)}{\cos(\theta+\lambda)} \leq P \leq w\frac{\sin(\alpha+\lambda)}{\cos(\theta-\lambda)}$$

   (b) Find the height at which a particle can rest inside a hollow rough sphere of radius 'a' if the coefficient of friction is $\frac{1}{\sqrt{3}}$.

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