(i) Printed Pages : 4] Roll No. ....

(ii) Questions : 8] **Sub. Code : 0 4 4 3**

Exam. Code : 0 0 0 5

B.A./B.Sc. (General) 5th Semester Examination

1127

MATHEMATICS

(Analysis–I)

Paper : I

Time : 3 Hours] **[Max. Marks : 30**

Note :– Attempt *five* questions in all, selecting at least *two* questions from each Section. All questions carry equal marks.

Section–A

1. (a) Prove that the set :

$$\{\ldots\ldots\ldots\ 2^{-3},\ 2^{-2},\ 2^{-1},\ 1,\ 2^{1},\ 2^{2},\ 2^{3},\ \ldots\ldots\ldots\}$$

is countable.

(b) If $0 < x < 1$, then show that :

$$\frac{x}{1-x} \geq \log(1-x)^{-1} \geq x$$

2. (a) If $f : [a, b] \to R$ is a monotonic function, then it is integrable on $[a, b]$.

(b) If f be continuous function defined on $[a, b]$, then show that there exists a real number $\theta \in [0, 1]$ such that :

$$\int_a^b f(x)dx = (b-a)f\{a+\theta(b-a)\}$$

3. (a) If f is bounded and integrable in $[a, b]$, then $|f|$ is also bounded and integrable in $[a, b]$ and :

$$\left|\int_a^b f\,dx\right| \le \int_a^b |f|\,dx$$

(b) Show that :

$$\int_0^1 x^{-1/3}(1-x)^{-2/3}(1+2x)^{-1}\,dx$$

$$= \frac{1}{(9)^{1/3}}\beta\left(\frac{2}{3}, \frac{1}{3}\right)$$

by substituting $\frac{x}{1-x} = \frac{az}{1-z}$, where a is constant suitably selected.

4. (a) Prove that :

$$\Gamma(m)\Gamma\left(m+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2m-1}}(2m)$$

(b) Evaluate :

$$\int_0^{\pi/2}(\sin x)^{2/3}(\cos x)^{-1/2}dx$$

Section–B

5. Examine the convergence of the following :

(a) $\int_0^{\infty}\left(\frac{1}{1+x}-e^{-x}\right)\frac{dx}{x}$

(b) $\int_a^b \frac{dx}{(x-a)\sqrt{b-x}}$.

6. (a) Let $\phi(x)$ be bounded and monotonic in $[a, \infty)$ and tends to 0 as $x \to \infty$. Let $\int_a^t f(x)\,dx$ be bounded for all $t \geq a$. Then prove that

$$\int_a^{\infty} f(x)\phi(x)\,dx$$

is convergent at ∞.

(b) Show that :

$$\int_e^{\infty} \frac{\log x \sin x}{x} dx$$

is convergent.

7. (a) Discuss the convergence of :

$$\int_0^1 \frac{\log x}{1-x^2} dx$$

(b) Show that $\int_0^{\infty} \frac{\sin ax \sin bx}{x} dx$ converges to $\frac{1}{2}\log\left(\frac{a+b}{a-b}\right)$ where a and b are +ve reals.

8. By applying rule of differentiation under integral sign, prove the following :

(a) $\int_0^{\infty} \frac{e^{-xy} \sin x}{x} dx = \cot^{-1} y, y > 0$

(b) $\int_0^{\pi/2} \frac{\log(1 + b\sin^2 x)}{\sin^2 x} dx = \pi[\sqrt{1+b} - 1]$.