(i) Printed Pages : 3] Roll No. ....

(ii) Questions : 8] **Sub. Code : 0 4 4 4**

Exam. Code : 0 0 0 5

B.A./B.Sc. (General) 5th Semester Examination

1127

MATHEMATICS

(Modern Algebra)

Paper : II

Time : 3 Hours] **[Max. Marks : 30**

Note :– Attempt *five* questions in all, selecting at least *two* questions from each Section. All questions carry equal marks.

Section–A

1. (a) If $a, b \in G$ such that $ab = ba$ where G is a group, and $(0(a), o(b)) = 1$ then prove that :

$$0(ab) = 0(a)\,0(b)$$

(b) Prove that the centre Z(G) of a group G is a normal subgroup of G.

2. (a) Prove that every quotient group of a cyclic group is cyclic.

(b) Let H and K be two subgroups of a group G. Show that any coset relative to $H \cap K$ is the intersection of a coset relative to H with a coset relative to K.

3. (a) Let G and G′ be two groups. If $f : G \rightarrow G'$ is a homomorphism, show that the kernal of f is a normal subgroup of G.

(b) Prove that if for a group G, $f : G \rightarrow G$ is given by $f(x) = x^3$, $x \in G$ is an automorphism then G is abelian.

4. (a) If G be a non-abelian group such that $O(G) = p^3$, where p is a prime number, then prove that $O(Z) = p$, where z is centre of G.

(b) Prove that A_4, alternating group of order 4, has no subgroup of order six.

Section–B

5. (a) Prove that a commutative ring R with identity $1 \neq 0$ is an integral domain iff the cancellation laws hold for multiplication.

(b) If R is a ring in which $x^2 = x \ \forall \ x \in$ R. Prove that R is a commutative ring of characteristics 2.

6. (a) Prove that union of two left (right) ideals of a ring is an ideal iff one is contained in the other.

(b) Show that any ideal of Z is maximal iff it is generated by some prime element.

7. (a) Show that M $\neq \{0\}$ is a maximal ideal of a ring R iff for any ideal I of R, either I $\subseteq$ M or I + M = R.

(b) Show that isomorphic images of an integral domain is an integral domain.

8. (a) If R is an integral domain, then prove that R[x] is also integral domain.

(b) For any ring R, show that :

$$R[x] \Big| < x > \ \cong R$$