(i) Printed Pages : 4] Roll No. .....

(ii) Questions : 8] **Sub. Code : 0 4 4 5**

Exam. Code : 0 0 0 5

B.A./B.Sc. (General) 5th Semester Examination

1127

MATHEMATICS

(Probability theory)

Paper : III

Time : 3 Hours] **[Max. Marks : 30**

Note :– (i) Attempt *five* questions in all, selecting at least *two* questions from each Section.

Section–A

1. (a) If p and q are chosen randomly from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ with replacement, determine the probabilities that the roots of the equation $x^2 + px + q = 0$ are real.

   (b) Fatima and John appear in an interview for two vacancies in the same post. The probability of Fatima's selection is $\frac{1}{7}$ and that of John's selection is $\frac{1}{5}$. What is the probability that

(i) both of them will be selected ? (ii) only one of them is selected ? (iii) None of them will be selected ? (3,3)

2. (a) Of the students in a college, it is known that 30% has 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who has 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has A grade. What is the probability that the students have 100% attendance.

(b) A bowl contains 10 balls of same size and shape, one of the balls is red. Balls are drawn one by one at random and without replacement, until the red ball is drawn. Find the p.m.f. and c.d.f. of random variable X, the number of trials needed to draw the red chip. (3,3)

3. (a) The distribution function for a random variable

$$X \text{ is } F(x) = \begin{cases} 0 & , \quad x < 0 \\ 1 - e^{-2x} & , \quad x \geq 0 \end{cases}.$$

Find (i) the probability density function of X (ii) the probability that $X > 2$ (iii) the probability that $-3 < X \leq 4$.

(b) Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards. Find the mean and variance of the number of red cards. (3,3)

4. (a) A die is thrown 6 times. 'If getting an odd number is a success, what is the probability of (i) 5 successes ? (ii) at least 5 successes ? (iii) at most 5 successes ?

(b) If X has Poisson distribution with parameter m, then show that $P(X = \text{even}) = \frac{1}{2}(1+e^{-2m})$. (3,3)

Section–B

5. (a) Prove that the mean and variance of uniform distribution on $[a, b]$ are $\frac{a+b}{2}$ and $\frac{(b-a)^2}{12}$ respectively.

(b) Find the constant c so that

$$f(x) = \begin{cases} cx(3-x)^4, & 0 < x < 3 \\ 0, & \text{elsewhere} \end{cases}$$

is a p.d.f. (3,3)

6. (a) If the level of education among adults in a certain region is normally distributed with mean 8 and S.D. 5, what is the probability that in a sample of 100 adults, there is an average level of education (i) between 10 to 14 years (ii) more than 14 years.

(b) Two dice are rolled, Let X denotes the sum on the two faces and Y the absolute value of their difference. Assuming that the dice are fair, find (i) $P[(x = 5) \cap (Y = 1)]$,

(ii) $P[(x = 7) \cap (Y \geq 3)]$,

(iii) $P(X = Y)$. (3,3)

7. (a) The joint probability density function of a bivariate random variable (X, Y) is given by

$$f(x,y) = \begin{cases} ce^{-(2x+2y)}, & x \geq 0, y \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Find (i) c (ii) marginal probability density function (iii) conditional probability density function.

(b) If X has gamma distribution with $\alpha = \frac{r}{2}$, $r \in N$ and $\beta > 0$, then show that $Y = \frac{2X}{\beta}$ is $\chi^2(r)$. (3,3)

8. (a) Let $f(x,y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0, & \text{elsewhere} \end{cases}$

Find (i) $E(Y|X = x)$, $E(X|Y = y)$ (ii) Var $(Y|X = x)$.

(b) Let X and Y have a bivariate normal distribution with parameters $\mu_X = 2{\cdot}8$, $\mu_Y = 110$, $\sigma_X^2 = 0{\cdot}16$, $\sigma_Y^2 = 100$ and $r = \frac{3}{5}$. Compute

(i) $P(106 < Y < 124)$

(ii) $P(106 < Y < 124 \mid X = 3{\cdot}2)$. (3,3)