(i) Printed Pages : 4] Roll No. ................................

(ii) Questions : 9] **Sub. Code : 3 7 0 2**

**Exam. Code : 0 4 7 2**

# M.Sc. 1st Semester Examination

# 1127

## PHYSICS

## (Mathematical Physics-I)

## Paper : PHY–6001

**Time : 3 Hours]** **[Max. Marks : 60**

*Note* :– Attempt *five* questions in all, selecting at least one question from each Unit-I to IV and Unit-V is compulsory.

### Unit–I

1. (a) State and prove Cauchy-Reimann condition theorem. 6

   (b) Use $f(re^{i\theta}) = R(r, \theta)e^{r\Theta(r, \theta)}$ in which $R(r, \theta)$ and $\Theta(r, \theta)$ are differentiable real functions of $r$ and $\theta$ and write Cauchy-Riemann conditions in polar form. 6

2. (a) Prove that $\int_0^\infty \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{4a^3}, a > 0.$ 6

   (b) Discuss briefly about dispersion relations. 6

## Unit–II

3. (a) Show that :

   (i) $x\delta(x) = 0$

   (ii) $x\delta'(x) = -\delta(x)$

   (iii) $\delta(ax) = \frac{1}{|a|}\delta(x)$. 6

   (b) Show that $\int_{-1}^{1}(1-x^2)^n dx = 2^{2n+1}\frac{n!n!}{(2n+1)!}$. 6

4. (a) Show that the following sequence is a form of delta function :

$$\delta_n(x) = \begin{cases} 0 & x < -1/2n \\ n & -1/2n < x < 1/2n \\ 0 & x > 1/2n \end{cases}$$ 6

   (b) Define Beta and Gamma functions and derive relationship between beta and gamma functions. 6

## Unit–III

5. (a) Prove that the condition for the solutions of $n$th order differential equation to be linearly independent is that the Wronskian is nonzero. 6

   (b) Solve the following differential equation using power series method :

$$y''(x) - 2xy'(x) + 2ny(x) = 0,$$

   where $n$ is a constant. 6

6. (a) Use method of sparation of variables to split the Differential equation into ordinary differential equations :

$$\frac{-\hbar^2}{2\mu}\left\{\frac{1}{r^2\sin\theta}\left(\sin\theta\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)\right)+\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)\right.$$

$$\left.+\frac{1}{\sin\theta}\frac{\partial^2\psi}{\partial\phi^2}\right\}+V\psi=E\psi. \qquad 6$$

(b) Find the second solution of the differential equaiton $y''(x) + \omega^2 y(x) = 0$ if the first solution is given by $f_1(x) = a \sin \omega x$. 6

## Unit–IV

7. (a) Starting from recursion relation :

$$P'_{n+1}(x) = xP'_n(x) + (n+1)P_n(x).$$

Show that :

$$I_{l,n} = \int_0^1 x^l P_n(x)dx = \frac{l}{l-n} I_{l-1,\, n+1}. \qquad 6$$

(b) Find the integral representation of Bessel functions. 6

8. (a) Starting from the series form of Bessel function of order $v$, prove the following recursion relations :

(i) $J_{v-1}(x) + J_{v+1}(x) = \frac{2v}{x} J_v(x)$

(ii) $-J_{v+1}(x) + J_{v-1}(x) = 2J'_v(x)$. 6

(b) State and prove orthogonality condition for Hermite polynomials. 6

**Unit–V**

9. (a) Discuss analytic continuation. 3

(b) List the generating functions of :

(i) Legendre polynomials

(ii) Hermite polynomials

(iii) Lagurerre polynomials 3

(c) State Cayley-Hamilton theorm. 2

(d) How do we find the singularities of a differential equation ? 2

(e) Evaluate :

$$I = \int_{-\infty}^{\infty} \frac{dx}{1+x^2}.$$ 2