(i) Printed Pages: 3 Roll No. .....

(ii) Questions : 8 Sub. Code : 0 1 4 6

Exam. Code : 0 0 0 2

B.A./B.Sc. (General) 2nd Semester

1048

MATHEMATICS

Paper : II Calculus-II

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— (1) Attempt **five** questions in all, selecting at least **two** questions from each section.

(2) Each questions carries **6** marks.

SECTION—I

1. (i) Show that the line joining the two points of inflexion of the curve :

$$y^2(x - a) = x^2(x + a),\ x \neq \pm a$$

subtends an angle $\pi/3$ at the origin.

(ii) Trace the curve $y^2 = (x + 1)^3$. 3,3

2. (i) Find the asymptotes of the curve :

$$x^2y + xy^2 + 2x^2 - 2xy - y^2 - 6x - 2y + 2 = 0$$

and show that they cut the curve in at most three points which lie on the straight line $2x - 3y - 4 = 0$.

(ii) Determine the position and nature of the double points on the curve :

$$x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0.$$ 3,3

3. (i) Define circle of curvature. Find the equation of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$.

(ii) Show that the points of intersection of the curve $xy(x^2 - y^2) - 25x^2 - 9y^2 + 144 = 0$ and its asymptotes lie on ellipse whose eccentricity is 4/5. 3,3

4. (i) If C_o, C_p denote the lengths of chord of curvatures of the cardioid $r = a(1 + \cos\theta)$ along and perpendicular to the radius vector through any point respectively. Prove that :

$$3(C_o^2 + C_p^2) = 8aC_o.$$

(ii) Find the interval in which the curve $y = (x^2 + 4x + 5)e^{-x}$ is concave upwards or downwards. 4,2

SECTION—II

5. (i) If $\int_0^{\pi/4} \tan^n x\,dx$, show that, for $n > 1$, $I_n + I_{n-2} = \frac{1}{n-1}$.

Hence deduce the value of I_3.

(ii) Evaluate $\int \cosh^{-1}\left(\frac{1+x^2}{1-x^2}\right)dx$. 6

6. (i) Find the length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ measured from (0, a) to any point (x, y).

(ii) Find the volume of the solid obtained by revolving the area included between the curves $y^2 = x^3$ and $x^2 = y^3$ about X-axis. 6

7. (i) Find the surface area of the solid obtained by revolving the curve $y = 2x + 1 + \frac{1}{x^2}$ about x-axis for $1 \leq x \leq 2$.

(ii) Use Simpson's rule with n = 4 to approximate $\int_{-1}^{1} (x^3 + 1)\,dx$. Also find the error. 3,3

8. (i) Evaluate :

$$\lim_{n \to \infty} \frac{1}{n^{16}} (1^{15} + 2^{15} + \ldots\ldots + n^{15}).$$

(ii) Derive the reduction formula for $\int x^n \sin(ax)\,dx$. Hence evaluate $\int_0^{\pi/2} x^3 \sin(x)\,dx$. 2,4