[Total No. of (i) Printed Pages 4 (ii) Questions 8]

Sub Code : 0341 (1048) **Exam Code :** 0004

Exam : B.A./B.Sc. (General) 4th Semester

Subject : Mathematics

Paper : Paper-I Advanced Calculaus-II

Time : 3 Hours ***Maximum Marks : 30***

Note: Attempt **five** questions in all, selecting at least **two** questions from each unit. **All** questions carry equal marks.

UNIT-I

1. (a) Prove that the sequence $\{a_n\}$ where

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \ldots\ldots + \frac{1}{2n}$$ is convergent. 3

(b) Prove that the sequence $\{a_n\}$ defined by $a_1 = \sqrt{2},\ a_n = \sqrt{2a_{n-1}}$ converges to 2. 3

2. (a) State and prove squeez principle.

(b) If $0 < s_1 < s_2$ and $s_n = \dfrac{2s_{n-1} + s_{n-2}}{s_{n-1} + s_{n-2}}$ show that $\{s_n\}$ converges to $\dfrac{3s_1 s_2}{2s_1 + s_2}$

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3. (a) Show that $\lim\limits_{n\to\infty}\frac{1}{n}\left(1+2^{1/2}+3^{1/3}+......+n^{1/n}\right)=1$ 3

(b) Show that the sequence $\{a_n\}$, where $a_n = 1+\frac{1}{2}+\frac{1}{3}+.........+\frac{1}{n}$ does not converge, by showing that it is not a Cauchy sequence. 3

4. (a) Show that f(x) = sin x is uniformly continuous on $\left[0, \frac{\pi}{2}\right]$ 3

(b) Show that the function f defined by :

$$f(x)=\begin{cases}1 & \text{if } x \text{ is rational}\\ 0 & \text{if } x \text{ is irrational}\end{cases}$$

is discontinuous every where. 3

UNIT-II

5. (a) Discuss the convergence or divergence of the series $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+........$ 3

(b) Use Cauchy's condensation test to show that $\sum \frac{1}{np}$, $p > 0$ converges if $p > 1$ and diverges if $p \leq 1$. 3

6. (a) Discuss the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{x^n}{3^n n^2}$ 3

(b) Discuss the convergence of the series $\frac{x}{x+1} + \frac{x^2}{x+2} + \frac{x^3}{x+3} + \ldots\ldots, x > 0$ 3

7. (a) Examine the convergence or divergence of the series $\frac{1^2}{2^2} + \frac{1^2.3^2}{2^2.4^2} x + \frac{1^2.3^2.5^2}{2^2.4^2.6^2} x^2 + \ldots\ldots\ldots, x>0.$

3

(b) Show that the series $1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} +$ .... is convergent.

3

8. (a) Show that the series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$........ is convergent for $-1 < x \leq 1$. 3

(b) Find the sum of the series $1+\frac{1}{3}+\frac{1}{5}-\frac{1}{2}+\frac{1}{7}+\frac{1}{9}+\frac{1}{11}-\frac{1}{4}+\ldots\ldots\ldots$.