[Total No. of (i) Printed Pages 4 (ii) Questions 8]

Sub Code : 0342 (1048) **Exam Code :** 0004

Exam : B.A./B.Sc. (General), 4th Semester

Subject : Mathematics

Paper : Paper-II : Differential Equations-II

Time : 3 Hours ***Maximum Marks : 30***

Note: Attempt **five** questions in all, selecting at least **two** questions from each unit.

UNIT-I

1. (a) Solve $y'' + (x-1)^2 y' - 4(x-1)y = 0$ about $x = 1$, in series. 3

(b) Solve in series the differential equation

$y'' - 2xy' + 2ny = 0$ 3

2. (a) Prove that $\frac{d}{dx}\left(x^n J_n(x)\right) = x^n J_{n-1}(x)$ 3

P.T.O.

(b) Using $\sin(x\sin\theta) = (2\sin\theta)J_1 + (2\sin 3\theta)J_3 + (2\sin 5\theta)J_5 + \ldots..$

show that $x \cos x = 2\left(1^2 J_1 - 3^2 J_3 + 5^2 J_5 \ldots\ldots\right)$

3

3. (a) Show that $\int_{-1}^{1} x^k P_n(x)\, dx = 0$,

Where k is an integer less than n.

3

(b) Prove $(2n+1)x P_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$, for $n \geq 1$)

3

4. (a) Solve Lagrange's linear equation for the general solution : 3

$P + 3q = 5z - \tan(3x - y)$

(b) Find the integral surface of $x^2 p + y^2 q = -z^2$ which passes through the hyperbola

$x + y = xy,\ z = 1.$ 3

UNIT-II

5. (a) State and prove existence theorem of Laplace transform : 3

(b) Evaluate $L\left(\int_0^t e^{-z}\cos z\, dz\right)$ 3

6. (a) Evaluate $L^{-1}\left(\frac{2s}{s^4+s^2+1}\right)$ 3

(b) Find inverse Laplace transform of

$\frac{1}{s^3(s+1)}$ 3

7. (a) State and prove Convolution Theorem to find inverse Laplace transform of the product of two functions . 3

(b) Apply Convolution theorem to evaluate

$L^{-1}\left(\frac{s^2}{s^4-a^4}\right)$ 3

8. (a) Solve the integral equation

$$y(t) = e^{-t} - 2\int_0^t y(u)\cos(t-u)\,du \qquad 3$$

(b) Solve : $\dfrac{dx}{dt} + 5x - 2y = t$ 3

$$\frac{dy}{dt} + 2x + y = 0$$

When $x(0) = y(0) = 0$