(i) Printed Pages: 3 Roll No. ....

(ii) Questions : 8 **Sub. Code : 0 5 4 2**

Exam. Code : 0 0 0 6

B.A./B.Sc. (General) 6th Semester

1048

MATHEMATICS

Paper–II : Linear Algebra

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** questions from each unit. All questions carry equal marks.

UNIT—I

I. (a) Prove that the necessary and sufficient condition for non-empty subset W of a vector space V(F) to be a subspace of V is that $\alpha x + \beta y \in W$ for $\alpha, \beta \in F$ and $x, y \in W$.

(b) Let $V = \{A \mid A = [a_{ij}]_{n \times n}, a_{ij} \in R\}$ be a vector space over reals. Show that W, the set consisting of all the symmetric matrices is a subspace of V. 3,3

II. (a) If W_1 is a subspace of vector space V(F), then prove that $\exists$ a subspace W_2 of V(F) such that $V = W_1 \oplus W_2$.

(b) Show that linear transformation $T : R^2 \to R^3$ defined by $T(x, y) = (x, x - y, x + y)$ is one-one but not onto. 3,3

III. (a) State and prove *Extension Theorem.*

(b) Show that dimension of the vector space $Q(\sqrt{2}, \sqrt{3})$ over Q is 4. 4,2

IV. (a) State and prove *Rank–Nullity Theorem.*

(b) Let T be a linear operator on R^3 defined by

$T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$.

Show that T is invertible and find T^{-1}. 4,2

UNIT—II

V. (a) Let T be a linear operator on R^3 defined by

$T(x, y, z) = (2y + z, x - 4y, 3x)$.

(i) Find the matrix of T relative to the basis

$B = \{(1, 1, 1); (1, 1, 0), (1, 0,0)\}$

(ii) Also verify that [T ; B] [v, B] = [T(v); B] for any vector $v \in R^3$.

(b) Let A be a non-singular matrix over a field F and $\lambda \in F$ be an eigen value of A. Prove λ^{-1} is an eigen value of A^{-1}. 4,2

VI. (a) Find the linear mapping $T : R^2 \to R^3$ determined by the matrix

$A = \begin{bmatrix} 0 & 2 \\ 1 & -1 \\ 2 & 3 \end{bmatrix}$ w.r.t., the ordered basis $B_1 = \{(1, 2), (0, 3)\}$

and $B_2 = \{(1, 1, 0), (0, 1, 1), (1, 1, 1)\}$ for R^2 and R^3 respectively.

(b) Consider the following basis of $R_1^2 B_1 = \{(1, 0), (0, 1)\}$ and $B_2 = \{(1, 2), (2, 3)\}$. Find the transition matrices P and Q from B_1 to B_2 and B_2 to B_1 respectively. Verify $Q = P^{-1}$. 3,3

VII. (a) Let $T : R^3 \rightarrow R^3$ be L. T. which is represented in the standard ordered basis by matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$. Prove that T is diagonalizable. 4,2

(b) Find characteristic equation and characteristic roots of zero and identity matrices of order n.

VIII. (a) Prove that any two characteristic vectors corresponding to two distinct characteristic roots of a unitary matrix are orthogonal.

(b) Let a linear operator $T : R^3 \rightarrow R^3$ be defined as

$$T(x, y, z) = (x + y, y + z, z).$$

Find Characteristic Polynomial and minimal polynomial of T. 3,3