(i) **Printed Pages : 3** **Roll No. .....**

(ii) **Questions : 8** **Sub. Code :** | 0 | 0 | 4 | 3 |

Exam. Code : | 0 | 0 | 0 | 1 |

B.A./B.Sc. (General) 1st Semester

1125

MATHEMATICS

Paper : II : Calculus

Time Allowed : 3 Hours] **[Maximum Marks : 30**

Note :– (1) Attempt **five** questions, selecting at least **two** questions from each Unit.

(2) Each question will carry 6 marks.

UNIT–I

I. (a) Between any two distinct real numbers, there exist infinitely many real numbers.

(b) If $|x - 2| < 3$ then prove that :

$$\frac{-18}{7} < \frac{x^2 - 1}{x + 2} < 6. \qquad 3,3$$

II. (a) Show that $\lim_{x \to 1} \frac{1}{x - 1}$ does not exist.

(b) Evaluate $\lim_{x \to \infty} \frac{\sqrt{3x^2 + 4x - 5} - \sqrt{2x^2 + 3}}{4x + 7}$. 3,3

III. (a) If f is continuous at x = a, then | f | is also continuous at x = a.

(b) Find the value of a and b so that :

$$\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3}$$ exists and is equal to 1. 2,4

IV. (a) Evaluate : $\lim_{x \to 0} \frac{(1+x)^{1/x} - e}{x}$.

(b) Evaluate : $\lim_{x \to 0} (\sin x)^{\tan x}$. 3,3

UNIT–II

V. (a) Differentiate $y = \tanh^{-1}(\sinh x) + \coth^{-1}(x^3) - \text{sech}^{-1}(\cos x)$ w.r.t x.

(b) Use Cauchy's mean value theorem to evaluate

$$\lim_{x \to 1} \frac{\cos \frac{\pi}{2}}{\log 1/x} x.$$ 3,3

VI. (a) Use Mean Value Theorem to prove the following :

$$\pi/6 + \frac{2x-1}{\sqrt{3}} \leq \sin^{-1} x \leq \pi/6 + \frac{2x-1}{\sqrt{1-x^2}} \text{ for } 1/2 \leq x < 1.$$

(b) Let f be a function defined on [0 , x] such that :

(i) $f, f', f'', --, f^{(n-1)}$ are continuous in [0 , x]

(ii) $f^{(n)}$ exists in (0, x). Then there exists at least one θ, $0 < \theta < 1$ such that :

$$f(x) = f(0) + x f'(0) + \frac{x^2}{\lfloor 2} f''(0) + ... + \frac{x^{n-1}}{\lfloor n-1} f^{n-1}(0)$$

$$+ \frac{x^n}{\lfloor n} f^n(\theta x)$$ 3,3

VII. (a) Find n^{th} derivative of :

$y = e^{3x} \sin^2 2x$

(b) If $y = (\sin^{-1}x)^2$ then

$(1 - x^2)\, y_{n+2} - (2n + 1)\, x\, y_{n+1} - n^2 y_n = 0$ 3,3

VIII. (a) Find $\frac{d}{dx}[(x)^{\sin hx} + (\cos hx)^x]$

(b) Prove by Rolle's Theorem that equation $4x^3 - 6x^2 + 4x - 1 = 0$ has at least one real root in (0,1). 3,3