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# B.A./B.Sc. (General) 4th Semester Examination

# 1047

## MATHEMATICS

## Paper : I (Advanced Calculus–II)

**Time : 3 Hours]** **[Max. Marks : 30**

*Note* :– Attempt *five* questions in all, selecting at least *two* questions from each Unit. All questions carry equal marks.

### Unit–I

1. (a) Prove that the sequence $\{a_n\}$, where $a_n = \dfrac{2n-1}{3n+5}$ converges to $\dfrac{2}{3}$.

(b) If $s_1$ and $s_2$ are positive and $s_{n+1} = \sqrt{s_n s_{n-1}}$, prove that sequences $s_1, s_3, s_5, \ldots$ ; $s_2, s_4, s_6, \ldots$ are the one increasing and the other decreasing and their common limit is $(s_1 s_2^2)^{1/3}$.

2. (a) Show that the sequence $\{a_n\}$, where $a_1 = 1$ and $a_{n+1} = \sqrt{6+a_n}$ converges to 3.

(b) State and prove Cauchy's second theorem on limits.

3. (a) Show that the sequence $\{a_n\}$, where :

$$a_n = 1+\frac{1}{3}+\frac{1}{5}+....+\frac{1}{2n-1}$$

is not convergent. Prove that $\{a_n\}$ diverges to $\infty$.

(b) Show that :

$$\lim_{n\to\infty} n^{1/n} = 1$$

4. (a) Show that the function $f$ defined by :

$$f(x) = \begin{cases} x, & \text{if } x \text{ is irrational} \\ 0, & \text{if } x \text{ is rational} \end{cases}$$

is continuous only at $x = 0$.

(b) Show that $f(x) = \cos x$ is uniformly continuous on $\left[0, \frac{\pi}{2}\right]$.

## Unit–II

5. (a) Show that the alternating series :

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots..$$

is convergent.

(b) Discuss the convergence or divergence of the following series :

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots...$$

6. (a) Show that the sereis $\sum_{n=1}^{\infty} \frac{1}{n^p}, p > 1$ converges and its sum lies between $\frac{1}{p-1}$ and $\frac{p}{p-1}$.

(b) Examine the convergence or divergence of

$$\sum_{n=1}^{\infty} e^{\sqrt{n}} r^n, \quad r > 0$$

7. (a) Discuss the convergence or divergence of the series :

$$1+\frac{1}{2}.\frac{a}{b}+\frac{1.3}{2.4}.\frac{a(a+1)}{b(b+1)}+\frac{1.3.5}{2.4.6}.\frac{a(a+1)(a+2)}{b(b+1)(b+2)}+....,$$

$a > 0,\ b > 0$

(b) Examine the convergence or divergence of the series $\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}x^n, x>0$.

converges if $\beta > \alpha$ and diverges if $\beta \leq \alpha$.

8. (a) Prove that the series $x-\frac{x^3}{3}+\frac{x^4}{5}-\frac{x^7}{7}+.......$ is convergent for $-1 \leq x \leq 1$.

(b) Assuming that $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$, find the value of $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$.