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Exam. Code : 0 0 0 4

B.A./B.Sc. (General) 4th Semester Examination

1047

MATHEMATICS

Paper : II (Differential Equations–II)

Time : 3 Hours] [Max. Marks : 30

Note :– Attempt *five* questions in all, selecting at least *two* from each Unit.

Unit–I

1. (a) Solve in series the differential equation $y'' - xy' + y = 0$.

(b) Solve in series the differential equation :

$$(x + x^2)\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} - y = 0 \qquad 3,3$$

2. (a) For integral value of n, prove that :

$$J_{-n}(x) = (-1)^n J_n(x)$$

(b) Prove that :

$$J_n(x) = \frac{1}{2\pi}\int_0^{2\pi} \cos(x\sin\theta - n\theta)\,d\theta$$ 3,3

3. (a) If $m \neq n$ then show that :

$$\int_{-1}^{1} P_n(x) P_m(x)\,dx = 0$$

(b) Show that :

$$\int_{-1}^{1} x^m P_m(x)\,dx = \frac{2^{m+1}(\lfloor m)^2}{\lfloor 2m+1}$$ 3,3

4. (a) Find the general solution of the Lagrange's linear equation :

$$z\,(xp - yq) = y^2 - x^2$$

(b) Find integral surface of the differential equation $(y - z)\,p + (z - x)\,q = x - y$ which passes through the line $y = 2x,\ z = 0$. 3,3

Unit–II

5. (a) State and prove Linearity property of Laplace transform.

(b) Find Laplace transform of $\frac{\cos\sqrt{t}}{\sqrt{t}}$. 3,3

6. (a) Prove that :

$$\int_0^\infty e^{-tx^2} dx = \frac{1}{2}\sqrt{\frac{x}{t}}$$

(b) Evaluate :

$$L^{-1}\left(\frac{1}{s}\log\left(1+\frac{1}{s^2}\right)\right)$$ 3,3

7. (a) Evaluate :

$$L^{-1}\left(\frac{s^2-2a^2}{s^4+4a^4}\right)$$

(b) Apply convolution theorem to evaluate :

$$L^{-1}\left(\frac{1}{(s+2)^2(s-2)}\right)$$ 3,3

8. (a) Solve the initial value problem $X'' - 3X' + 2X = 1-e^{2t}$ where $X(0) = 1$, $X'(0) = 0$.

(b) Solve $X'' + Y'' + 5X - 3Y = 0$ and $Y'' + 3Y - 2X = 0$ where $X(0) = 0$, $Y(0) = 0$, $X'(0) = 2$, $Y'(0) = 3$. 3,3