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B.A./B.Sc. (General) 6th Semester Examination

1047

MATHEMATICS

Paper - II : (Linear Algebra)

Time : 3 Hours] [Max. Marks : 30

Note :– Attempt *five* question in all, selecting at least *two* from each.

Unit–I

1. (a) Let V be the set of positive real numbers. Let vector addition and scalar multiplication be defined as :

$x + y = xy$ and $\alpha x = x^{\alpha}$ for all $x, y \in$ V and $\alpha \in \mathbf{R}$.

Prove that $V_{\mathbf{R}}$ is a vector space.

(b) Show that the only non-trivial subspaces of $\mathbf{R}^2$ over field **R** are the lines through origin.

2. (a) If V_F is a finitely generated vector space, then prove that all the bases of V have the same number of elements.

(b) Let W be a subspace of $F_3[x]$ generated by polynomials :

$$f_1 = x^3 - 2x^2 + 4x + 1,$$

$$f_2 = 2x^3 - 3x^2 + 9x - 1,$$

$$f_3 = x^3 + 6x - 5$$

and

$$f_4 = 2x^3 - 5x^2 + 7x + 5.$$

Find a basis and dimension of W.

3. (a) Find a basis and dimension of the solution space of systems of homogeneous linear equations :

$$x + 2y - 4z + 3s - t = 0$$

$$x + 2y - 2z + 2s + t = 0$$

$$2x + 4y - 2z + 3s + t = 0$$

(b) State and prove Rank Nullity theorem.

4. (a) Find a linear transformation $T : \mathbf{R}^4 \to \mathbf{R}^3$ whose kernel is spanned by (0, 1, 2, 3) and (–1, 2, 3, 0).

(b) Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be defined as :

$$T(x, y, z) = (2x, 4x - y, 2x + 3y - z).$$

Prove that T is invertible and find T^{-1}.

Unit–II

5. (a) Let T be a linear operator on $\mathbf{R}^3$ defined by :

$$T(x, y, z) = (2x - 3y + 4z, 5x - y + 2z, 4x + 7y).$$

(i) Find matrix of T w.r.t standard basis $\mathcal{B}$ of $\mathbf{R}^3$

(ii) Verify that $[T; B]\,[\upsilon; \mathcal{B}] = [T(\upsilon); \mathcal{B}]$ $\forall\, \upsilon \in \mathbf{R}^3$.

(b) If the matrix of linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ relative to usual basis is $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix}$, then find the matrix of T relative to the basis $\mathcal{B}' = \{(0, 1, -1), (-1, 1, 0), (1, -1, 1)\}$

6. (a) Show that the matrices $A = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$ and $B = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix}$ are similar over C.

(b) Find eigen values and eigen vectors of

$$A = \begin{pmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{pmatrix}.$$

Also verify that geometric multiplicity of an eigen value cannot exceed its algebraic multiplicity.

7. (a) Let $T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ be a linear operator defined by $T(x, y, z) = (7x + 4y - z, 4x + 7y - z, - 4x - 4y + 4z)$.

Is T diagonalizable ?

(b) Find the characteristic polymomial of the operator $T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ defined by $T(x, y, z) = (x + y + z, x + y, x)$ and use Cayley-Hamilton theorem to find T^{-1}.

8. (a) Find minimal polynomial of the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$ Hence Find A^{-1}.

(b) Prove that characteristic roots of a linear operator $T : V \rightarrow V$ are independent of the choice of basis of V.