(i) Printed Pages : 3 Roll No. ....

(ii) Questions : 8 **Sub. Code : 0 3 3 9**

Exam. Code : 0 0 0 4

B.A. /B.Sc. (General) 4th Semester

1046

MATHEMATICS

Paper : I –Advanced Calculus-II

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :– Attempt **five** questions in all selecting at least **two** from each unit. All questions carry equal marks.

UNIT–I

1. (a) By definition, show that $\lim_{n\to\infty} \frac{3n}{n+5\sqrt{n}} = 3$.

(b) Let $\{x_n\}$ be a sequence of positive numbers such that $x_{n+1} = \frac{2a^2 x_n}{x_n^{\,2}+a^2}\ \forall n \in N, a>0$. Show that $\lim_{n\to\infty} x_n = a$.

2. (a) Prove that $\lim_{n\to\infty}\left(\frac{1}{(n+1)^{\frac{4}{3}}}+\frac{1}{(n+2)^{\frac{4}{3}}}+....+\frac{1}{(2n)^{\frac{4}{3}}}\right)=0$.

(b) Prove that the sequence $\{a_n\}$, where $a_n = 1+\frac{1}{2}+\frac{1}{3}+......+\frac{1}{n}$ is divergent.

3. (a) Prove that the sequence $\{a_n\}$, where $a_n = \left[\frac{(3n)!}{(n!)^3}\right]^{\frac{1}{n}}$ is convergent.

(b) If $\{a_n\}$ and $\{b_n\}$ are convergent sequences, then show by Cauchy's general principle of convergence that $\{a_n b_n\}$ is convergent.

4. (a) Let f be a function defined on (0,1) by :

$$f(x)=\begin{cases} 0, & \text{if } x \text{ is an irrational number} \\ \frac{1}{q}, & \text{if } x=\frac{p}{q}, p,q \in N, (p,q)=1 \end{cases}$$

Show that f is continuous at each irrational point and discontinuous at each rational point.

(b) Let $f: (0,\infty) \to R$ be a function defined by $f(x)=\frac{1}{x}$. Prove that f is uniformaly continuous on $[a,\infty)$, where $a>0$. Also show that f is continuous but not uniformly continuous on $(0,\infty)$.

UNIT–II

5. (a) State and prove Cauchy's condensation test for convergence of series.

(b) Show that the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ is convergent iff $p > 1$.

6. (a) Prove that $\lim_{n\to\infty}\left(1+\frac{1}{2}+\frac{1}{3}+......+\frac{1}{n}-\log n\right)$ exists and lies between 0 and 1.

(b) Discuss the convergence of the series :

$$x^2(\log 2)^m + x^3(\log 3)^m + x^4(\log 4)^m + ..., m>0.$$

7. (a) Examine the convergence or divergence of the series

$$\frac{1^2}{2^2}+\frac{1^2.3^2.}{2^2.4^2.}.x+\frac{1^2.3^2.5^2}{2^2.4^2.6^2}.x^2+......, x>0.$$

(b) Examine the convergence of the series :

$$\sum_{n=1}^{\infty}(-1)^n\left(\frac{1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}}{n}\right).$$

8. (a) Show that the series is conditionally convergent :

$$\frac{1}{\sqrt{2}+1}-\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{4}+1}-\frac{1}{\sqrt{5}+1}+......$$

(b) What rearrangement of the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-.....$ will reduce its sum to $\frac{1}{2}\log 2$?