(i) Printed Pages : 3 Roll No. ....

(ii) Questions : 8 **Sub. Code : 0 4 7 7**

Exam. Code : 1 0 3

B.A. /B.Sc. (General) 3rd Year

1046

MATHEMATICS

Paper : II: Abstract Algebra

Time Allowed : Three Hours] **[Maximum Marks : 60**

Note :– Attempt **five** questions in all, selecting at least **two** questions from each section.

SECTION-A

I. (a) Prove that a semi group in which both the equations ax =b and ya = b have a unique solution is a group. 6

(b) Prove that every subgroup of a cyclic group is cyclic. 6

II. (a) Prove that a group in which every element is its own inverse is abelian. 3

(b) Define index of a subgroup and prove that a subgroup of index 2 is normal. 4

(c) Prove that every finite cyclic group of order n is isomorphic to $\frac{z}{n^{z}}$. 5

III. (a) Prove that every group is isomorphic to a permutation group. 6

(b) If an ideal M of a commutative ring R with unity is maximal then prove that R/M is a field. Also prove converse part. 6

IV. (a) Let I and J be two ideals of a ring R then prove that $(I + J)/I \cong J/(I \cap J)$. 6

(b) Show that an ideal $< x >$ of Z[x] is a prime ideal but not a maximal ideal. 6

SECTION-B

V. (a) Let V_F be a vector space. Then prove that a finite subset $S = \{ x_1, x_2, ...x_n\}$ of non zero element of V is linearly dependent iff some element of S say x_i can be written as linear combination of other element of S. 6

(b) Prove that a linear independent subset of V can not contain zero vector. 3

(c) Check whether $W= \{ (x, y, z) ; ax + by + cz = 0\}$ is a subspace of R^3, where R^3 be a vector space over R and a, b, c, $\in$ R. 3

VI. (a) Prove that every subspace of a vector space has a direct summand. 6

(b) Find the linear transformation $T : R^4 \rightarrow R^3$ whose kernel is spanned by (0, 1, 2, 3) and (–1, 2, 3, 0). 6

VII. (a) Let T be a linear operator on R^3 defined by $T(x, y, z) = (x - 2y - z, y - z, x)$ show that T is invertible and find T^{-1}. 6

(b) Let $T : V \rightarrow V$ be a linear operator where V is finite dimensional vector space over F. If B is an ordered basis of V. Then for any $v \in V$ prove that $[T(V); B] = [T ; B] [V ; B]$. 6

VIII. (a) Prove that eigen vectors corresponds to distinct eigen values of linear operator are independent. 6

(b) Let a linear operator $T : R^3 \rightarrow R^3$ be defined as $T(x, y, z) = (x + y, y + z, z)$. Find the characteristic and minimal polynomial of T. 6