

1125

M.Sc. (Physics) First Semester  
PHY-6001: Mathematical Physics – I

Time allowed: 3 Hours

Max. Marks: 60

**NOTE:** Attempt five questions in all, including Question No. 9 (Unit-V) which is compulsory and selecting one question each from Unit I – IV.

x-x-x

UNIT –I

Q1. a) State and Prove Laurent Series expansion theorem. 6.

b) Evaluate  $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + a^2)(x^2 - b^2)}$ ,  $a, b > 0$ . 6.

Q2. a) Is the function  $u(x, y) = \sin x \cosh y$  harmonic? Find its conjugate function  $v(x, y)$ . 6.

b) Using calculus of residues prove that

$$\int_{-\infty}^{\infty} \frac{e^{ax}}{1 + e^x} dx = \frac{\pi}{\sin \pi a}, \quad 0 < a < 1 \quad 6.$$

Unit- II

Q3. a) Show that  $\delta_n(x) = \frac{n}{\pi} \left( \frac{1}{1 + n^2 x^2} \right)$  is a delta function in the limit. 6.

b) Evaluate the integral using beta function  $\int_0^2 \frac{x^6 dx}{(4 - x^2)^{3/2}}$  6.

Q4. a) Show that  $\delta'(x - x_0) = -\frac{\delta(x - x_0)}{(x - x_0)}$  6.

b) Using Weierstrass infinite product form, derive  $\Gamma(z)\Gamma(1 - z) = \frac{\pi}{\sin \pi z}$ . 6.

Unit- III

Q5. a) Write steps to find second linear independent solution of a homogeneous second order differential equation if one of its solution is known. 6.

b) Write Laplace equation in two dimension polar coordinates and separate it. 6.

Q6. a) Solve the Legendre differential equation using Frobenius method. 9.

b) What is the use of differential equations in physics. 3.

Unit-IV

Q7. a) Derive orthogonality relation for Associated Legendre Polynomials. 6.

b) Prove the following recurrence relation for Laguerre polynomials  
 $(n + 1)L_{n+1}(x) = (2n + 1 - x)L_n(x) - nL_{n-1}(x)$ . 6.

Q8. a) If  $n > -1$ , show that  $\int_0^x x^{-n} J_{n+1}(x) dx = \frac{1}{2^n n!} - x^{-n} J_n(x)$ . 6.

b) Derive the following recurrence relation for Legendre polynomials:  
 $(1 - x^2)P_n'(x) = (n + 1)xP_n(x) - (n + 1)P_{n+1}(x)$ . 6.

Unit -V

Q9. a) Write the three forms of Gamma function. 2.

b) How do we find the residue for a pole of order m. 2.

c) What are dispersion relations? How are they useful? 3.

d) Calculate first three Legendre polynomials using Rodrigue's formula. 3.

e) State generating functions for Hermite and Laguerre Polynomials. 2.

x-x-x