Exam. Code: 596
Sub. Code: 4139

1056

Certificate (Add-on Course)

Environment Auditing

Paper-II(B): Introductory Mathematics

(For Medical Stream)

Time Allowed: 3 hours

Max. Marks: 75

Note: *Attempt **five** questions in all.*

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I. a) If the coefficient of x^2 and x^3 in the expansion of $(3 + Kx)^9$ are equal, find the value of K.

b) In how many ways can 10 examination papers be arranged so that (i) the best and the worst papers never come together; (ii) the best and worst papers are always together?

c) Prove by the principle of mathematical induction:

$1 + 3 + 5 + \ldots\ldots + (2n-1) = n^2$. (5,5,5)

II. a) Let A and B be two sets such that

$n(A) = 18$, $n(A \cup B) = 39$ and $n(A \cap B) = 3$.

Find (i) $n(A-B)$; (ii) $n(B)$; (iii) $n(B-A)$

b) If $A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$, which of the following are relations from A to B? Give reasons to support your answer.

(i) $R_1 = \{(1,5), (1, 6), (2,4), (3,5)\}$

(ii) $R_2 = \{(1,4), (3,4)\}$

(iii) $R_3 = \{(1,5), (5,2), (1,4), (3,6), (2,4)\}$

c) Let $A = \{-2, -1, 0, 1, 2\}$ and $f : A \rightarrow Z$ be defined by $x^2 - 2x - 3$.

Find (i) the range of f, (ii) pre-images of -3 and 5. (5,5,5)

III. a) Determine whether 3 is a root of the equation

$$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 16}$$

b) Find the roots of the following equation :

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, \quad x \neq -4, 7$$

c) Find the value of K for which $Kx(x-2) + 6 = 0$ have two equal roots. (5,5,5)

IV. a) Show that $f(x) = \begin{cases} 5x - 4, & 0 < x \leq 1 \\ 4x^2 - 3x, & 1 < x < 2 \end{cases}$

is continuous at $x = 1$.

b) Show the $f(x) = x^2$ is differentiable at $x = 1$ and $f'(1)$.

c) If $y = x + \sqrt{x^2 + a^2}$, then prove that $\frac{dy}{dx} = \frac{y}{\sqrt{x^2 + a^2}}$ (5,5,5)

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V. a) If $f'(x) = \frac{x}{2} + \frac{2}{x}$ and $f(1) = 5/4$, find $f(x)$.

b) Evaluate

(i) $\int \frac{x^5}{1+x^6}\,dx$, (ii) $\int \frac{2x}{(2x+1)^2}\,dx$ (5,5,5)

VI. a) Prove that
$(\text{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$.

b) If $\sin\theta + \cos\theta = p$ and $\sec\theta + \text{cosec}\,\theta = q$, show that:
$q(p^2 - 1) = 2p$

c) Prove that
$$\frac{\text{Cos}\,(90^\circ - \theta)}{1 + \sin(90^\circ - \theta)} + \frac{1 + \sin(90^\circ - \theta)}{\cos(90^\circ - \theta)} = 2\sec(90^\circ - \theta)$$
(5,5,5)

VII. a) If $|\bar{a} + \bar{b}| = |\bar{a} - \bar{b}|$, find the angle between $\bar{a}$ and $\bar{b}$.

b) If a, b, c are three vectors of equal magnitude. The angle between each pair of vectors in $\pi/3$ such that $|\bar{a} + \bar{b} + \bar{c}| = \sqrt{6}$. Find the magnitude of $\bar{a}$.

(7,8)

VIII. a) Using the exponential series, prove that

$$\frac{1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots\ldots\infty}{1 + \frac{1}{3!} + \frac{1}{5!} + \frac{1}{7!} + \ldots\ldots\infty} = \frac{e^2 + 1}{e^2 - 1}$$

b) Show that

$$\log x = \frac{x-1}{x+1} + \frac{1}{2}\cdot\frac{x^2-1}{(x+1)^2} + \frac{1}{3}\cdot\frac{x^3-1}{(x+1)^3} + \ldots\ldots\infty$$ (8,7)

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