B.A./B.Sc. (General) 1ˢᵗ Semester

(2122)

MATHEMATICS

Paper–I : Plane Geometry

Time Allowed : Three Hours]   **[Maximum Marks : 30**

Note :—Attempt **FIVE** questions in all, selecting at least **TWO** questions from each unit. All questions carry equal marks.

UNIT—I

1. (a) Transform $5x^2 - 2xy + 5y^2 + 2x - 10y - 7 = 0$ to rectangular axes through $(0, 1)$ inclined at an angle of $\dfrac{\pi}{4}$ to the original axes.

   (b) Find the joint equation of two straight lines passing through $(1, 2)$ and perpendicular to lines

   $$3x^2 - 8xy + 5y^2 = 0.$$

2. (a) Show that the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + 2 = 0$ represent a pair of straight lines and find the angle between them.

(b) Prove that the angle between the lines joining the origin to the points of intersection of straight line $y = 3x + 2$ with the curve $x^2 + 2xy + 3y^2 + 4x + 8y - 11 = 0$ is $\tan^{-1}\left(\dfrac{2\sqrt{2}}{3}\right)$.

3. (a) Two circles each of radius 5 units touch each other at the point (1, 2). If the equation of their common tangent is $4x + 3y = 10$. Find the equation on the circles.

(b) Find the locus of the middle points of the chords of the circle $x^2 + y^2 + 6x + 2y - 10 = 0$ which subtends a right angle at the centre of the circle.

4. (a) Find the equation of the circle which passes through the points (2, 0), (0, 2) and is orthogonal to the circle $2x^2 + 2y^2 + 5x - 6y + 4 = 0$.

(b) Find the radical axis and limiting points of the co-axial system determined by the circles $x^2 + y^2 + 2x - 6y = 0$ and $2x^2 + 2y^2 - 10y + 5 = 0$.

UNIT—II

5. (a) Prove that the semi-latus rectum of the parabola is the harmonic means between the segments of a focal chord.

(b) Show that the locus of points such that two of the three normals from them to the parabola $y^2 = 4ax$ coincide is $12ay^2 = 4(x - 2a)^3$.

6. (a) Prove that the tangent and normal at any points of an ellipse bisect respectively the external and internal angles between the focal distances of the point.

   (b) Find the minimum angles between a pair of conjugate diameters of the ellipse $4x^2 + 9y^2 = 36$.

7. (a) Prove that the polar of the points $(-3, 4)$ w.r.t. the parabola $y^2 = 4x$ touches the ellipse $x^2 + 2y^2 = 3$ and find the co-ordinates of the point of contact.

   (b) Find joint equation of asymptotes to the hyperbola $3x^2 - 5xy - 2y^2 + 5x + 11y - 8 = 0$. Also find equation of its conjugate hyperbola.

8. (a) Prove that the locus of the middle points of normal chords of the rectangular hyperbola $x^2 - y^2 = a^2$ is $(y^2 - x^2)^3 = 4a^2x^2y^2$.

   (b) Identify the curve $4x^2 + y^2 - 8x + 2y + 1 = 0$. Also find foci and eccentricity.