(i) Printed Pages: 3 Roll No. ....

(ii) Questions : 8 **Sub. Code : 0 0 4 4**

Exam. Code : 0 0 0 1

B.A./B.Sc. (General) 1st Semester

(2122)

MATHEMATICS

Paper–II (Calculus–I)

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt **five** questions in all, selecting at least **two** questions from each Unit.

UNIT–I

1. (a) Prove that between any two distinct real numbers, there lie an infinite number of real numbers. 3

(b) If $|x - 3| < 2$, then prove that :

$$-\frac{9}{8} < \frac{x^2 - 2}{x + 3} < \frac{15}{4}$$ 3

2. (a) Solve the inequality $\frac{x-2}{x+2} < \frac{x+1}{x-1}$. 3

(b) Prove that $\underset{x \to a}{\text{Lt}} \frac{1}{x-a}$ does not exist. 3

3. (a) Let f be a continuous function defined in [a, b], $f(a) \neq f(b)$ and let k be any number lying between f(a) and f(b). Then prove that there exists $c \in (a, b)$ such that $f(c) = k$. 3

(b) Show that the function f defined by :

$$f(x) = \begin{Bmatrix} [x-2]+[2-x] & ; x \neq 2 \\ 0 & ; x = 2 \end{Bmatrix}$$

is discontinuous at $x = 2$. 3

4. (a) Evaluate $\underset{x \to 0}{\text{Lt}} \left[\frac{2(\cosh x - 1)}{x^2}\right]^{\frac{1}{x^2}}$. 3

(b) Show that $\underset{x \to 0}{\text{Lt}} \frac{x^2 \sin\frac{1}{x}}{\tan x}$ exists, but cannot be evaluated by L'Hospital rule. What is the limit ? 3

UNIT–II

5. (a) State and prove Cauchy's Mean Value theorem. 3

(b) Use Mean Value theorem to prove that :

$$\frac{x}{1+x} < \log(1+x) < x \text{ for } x > -1,\ x \neq 0.$$ 3

6. (a) Expand cos x in powers of $x - \frac{\pi}{2}$ by using Taylor's theorem. 3

(b) Show that $\frac{d}{dx}[\tanh(\log x)] = \frac{4x}{(x^2+1)^2}$. 3

7. (a) Using Maclaurin's theorem, expand the function $f(x) = e^{\sin x}$. 3

(b) Prove that $\tanh^{-1} x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right), |x| < 1$, and then find its derivative. 3

8. (a) If $x^3 + y^3 - 3axy = 0$, then show that :

$$y_2 = -\frac{2a^3xy}{(y^2 - ax)^3}$$ 3

(b) If $y = \frac{\log x}{x}$, then prove that :

$$y_n = \frac{(-1)^n \lfloor n}{x^{n+1}}\left[\log x - 1 - \frac{1}{2} - \frac{1}{3} - .... - \frac{1}{n}\right]$$ 3