24/12/2022 (morning)

(i) Printed Pages : 4 Roll No. ....

(ii) Questions : 8 **Sub. Code : 0 0 4 5**

Exam. Code : 0 0 0 1

B.A./B.Sc. (General) 1st Semester

(2122)

MATHEMATICS

Paper–III : Trigonometry & Matrices

Time Allowed : Three Hours] [Maximum Marks : 30

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** questions from each unit.

UNIT—I

1. (a) By taking $z = \cos\theta + i\sin\theta$, in the identity

$$z + z^3 + z^5 + ... + z^{2n-1} = \frac{z(1-z^{2n})}{1-z^2}, \text{ prove that}$$

(i) $\cos\theta + \cos 3\theta + \cos 5\theta + ... + \cos(2n-1)\theta = \dfrac{\sin 2n\theta}{2\sin\theta}$

(ii) $\sin\theta + \sin 3\theta + \sin 5\theta + ... + \sin(2n-1)\theta = \dfrac{1-\cos 2n\theta}{2\sin\theta}$

3

(b) Show that each primitive 12th root of unity satisfies $x^4 - x^2 + 1 = 0$. 3

2. (a) Use De Moivre's theorem to solve

$(4 + x)^5 - (4 - x)^5 = 0.$ 3

(b) If $\tan(x + iy) = \cosh(\alpha + i\beta)$, prove that

$\tanh\alpha \tan\beta = \operatorname{cosec} 2x \sinh 2y.$ 3

3. (a) For $z \in \mathbb{C}$, prove that

$\cosh^{-1} z = 2n\pi i \pm \log(z + \sqrt{z^2 - 1}),\ n \in \mathbb{Z}.$ 3

(b) If $\cosh^{-1}(x + iy) + \cosh^{-1}(x - iy) = \cosh^{-1}a$, show that $2(a - 1)x^2 + 2(a + 1)y^2 = a^2 - 1.$ 3

4. (a) Sum upto infinity the sereis

$\frac{1}{2}\sin\alpha + \frac{1.3}{2.4}\sin 2\alpha + \frac{1.3.5}{2.4.6}\sin 3\alpha + ...$ 3

(b) Prove that $\lim_{x \to 0} \frac{1}{x^2} \log\left(\frac{\tan^{-1} x}{x}\right) = \frac{-1}{3}.$ 3

UNIT—II

5. (a) Show that every Hermitian matrix A can be uniquely expressed as P + iQ, where P and Q are real symmetric and real skew symmetric matrices respectively. Also show that $A^\theta A$ is real iff $PQ = -QP$. 3

(b) Find the rank of the matrix $\begin{bmatrix} 1 & 3 & 1 & 3 \\ 2 & 8 & 5 & 10 \\ 1 & 7 & 7 & 11 \\ 3 & 11 & 7 & 15 \end{bmatrix}$. 3

6. (a) Find non-singular matrices P and Q such that PA Q is in normal form where $A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$ and hence find the rank of A. 3

(b) Discuss for all values of k, the system of equations

$$(3k - 8)x + 3y + 3z = 0$$

$$3x + (3k - 8)y + 3z = 0$$

$$3x + 3y + (3k - 8)z = 0$$

as regards the nature of solution. 3

7. (a) Solve the following system of equations, if consistent :

$$x + y - 2z + 4t = 5$$

$$2x + 2y - 3z + t = 3$$

$$3x + 3y - 4z - 2t = 1$$ 3

(b) Find the eigen values and the corresponding eigen vectors of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{bmatrix}$. 3

8. (a) Verify Cayley-Hamilton theorem for the matrix A and hence find its inverse, $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 0 & 2 & 1 \end{bmatrix}$. 3

(b) Let $A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$. Is A diagonalizable ? If it is, then find invertible matrix P such that $P^{-1}AP$ is a diagonal matrix. 3