(i) **Printed Pages : 3** **Roll No. ....**

(ii) **Questions : 8** **Sub. Code : 0 2 4 1**

Exam. Code : 0 0 0 3

B.A./B.Sc. (General) 3rd Semester

(2122)

MATHEMATICS

Paper : I (Advanced Calculus-I)

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** questions from each unit. All questions carry equal marks.

UNIT—I

1. (a) Let $f(x,y)=\begin{cases}\dfrac{x^2y}{x^4+y^2}, & x^2+y^2\neq 0\\ 0, & x=y=0.\end{cases}$

Prove that a straight line approach gives the limit zero, but the limit does not exist at (0, 0).

(b) Let $D = \{(x, y) : 0 < x < 1, 0 \leq y < 1\}$ and $f : D \rightarrow \mathbf{R}$ be defined by

$$f(x,y)=\begin{cases}1 & \text{if} \quad y\neq 0\\ 0 & \text{if} \quad y=0.\end{cases}$$

Show that f is not continuous at any point (a, 0), where $0 < a < 1$.

2. (a) If $\theta = t^n e^{-\frac{r^2}{4t}}$, find the value of n which will make

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}.$$

(b) If $x^y + y^x = (x + y)^{x+y}$, then by using partial derivatives, find $\frac{dy}{dx}$.

3. (a) State Young's theorem. Show that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$ for the function $f(x, y) = xy\left(\frac{x^2 - y^2}{x^2 + y^2}\right)$, where $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$.

(b) Show $f(x, y) = \cos x + \cos y$ is differentiable at every point of $\mathbf{R}^2$.

4. (a) Find the directional derivative of $f = \frac{y}{x^2 + y^2}$ in the direction of a vector which makes an angle of 30° with positive x-axis at point (0, 1).

(b) Prove that curl $\vec{f}$ = grad div $\vec{f} - \nabla^2 \vec{f}$.

UNIT—II

5. (a) State Euler's theorem on homogeneous functions of two variables. Verify it for $z = xy\, f\left(\frac{x}{y}\right)$.

(b) Expand $e^x \sin y$ in powers of x and y as far as the terms of third degree.

6. (a) If $u^3 + v + w = x + y^2 + z^2$, $u + v^3 + w = x^2 + y + z^2$, $u + v + w^3 = x^2 + y^2 + z$, prove that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = \frac{1 - 4(yz + zx + xy) + 16xyz}{2 - 3(u^2 + v^2 + w^2) + 27u^2v^2w^2}.$$

(b) Show that the functions $u = x + y + z$, $v = yz + zx + xy$, $w = x^3 + y^3 + z^3 - 3xyz$ are not independent of one another. Also find the relation between them.

7. (a) Find the envelope of a system of concentric and co-axial ellipses of constant area.

(b) Show that the evolute of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is

$$(x + y)^{\frac{2}{3}} + (x - y)^{\frac{2}{3}} = 2a^{\frac{2}{3}}.$$

8. (a) Find the maximum and minimum values of the function

$f(x, y) = \sin x + \sin y + \cos(x + y)$.

(b) Find the maximum value of the function $f(x, y, z) = xyz$ subject to the condition $xy + yz + zx = a$, where $x, y, z > 0$.