(i) **Printed Pages : 4** **Roll No. ....**

(ii) **Questions : 8**

Sub. Code : | 0 | 4 | 4 | 5 |

Exam. Code : | 0 | 0 | 0 | 5 |

B.A./B.Sc. (General) 5th Semester

(2122)

MATHEMATICS

Paper-III : Probability Theory

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** questions from each unit.

UNIT—I

1. (a) For any n events $A_1, A_2, ..., A_n$, Prove that

$$P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i).$$ 3

(b) An urn contains 10 white and 3 black balls, while another urn contains 3 white and 5 black balls. Two balls are drawn from the first urn and put into the second urn and then a ball is drawn from the second urn. What is the probability that it is white ? 3

2. (a) Let $f(x)=\begin{Bmatrix} Cx\,e^{-2x} & , & x \geq 0 \\ 0 & , & \text{elsewhere} \end{Bmatrix}$ be the probability density function of a random variable X. Then find :

(i) Constant C

(ii) The distribution function f(x)

(iii) $P(2 \leq X < 3)$

(iv) $P(X \geq 1)$. 3

(b) Let X have probability density function

$$f(x)=\begin{Bmatrix} \frac{1}{2}(x+1) & , & -1 < x < 1 \\ 0 & , & \text{elsewhere} \end{Bmatrix}.$$

Find mean and variance of X. 3

3. (a) A random variable X has probability density function given by $f(x)=\begin{Bmatrix} 2\,e^{-2x} & , & x \geq 0 \\ 0 & , & x < 0 \end{Bmatrix}$

Find :

(i) the moment generating function

(ii) first four moments about the origin. 3

(b) If $X \sim B(n, P)$. What is the distribution of $Y = n - X$?

3

4. (a) If X has Poisson distribution with parameter m, show that the distribution function of X is given by

$$F(x) = \frac{1}{\Gamma(x+1)} \int_m^{\infty} e^{-t} t^x \, dt \; ; \; x = 0, 1, 2, \ldots$$ 3

(b) If $X \sim B(n, P)$, show that $E\left(\frac{X}{n}\right) = P$ and

$$V\left(\frac{X}{n}\right) = \frac{P(1-P)}{n}.$$ 3

UNIT—II

5. (a) A random variable X has exponential distribution with parameter d = 3. Find :

(i) $P(X \geq 4)$

(ii) Standard deviation

(iii) Coefficient of variation. 3

(b) Define uniform random variable and show that mean deviation about mean of uniform distribution on [a, b] is $\frac{b-a}{4}$. 3

6. (a) A random variable X has probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \; -\infty < x < \infty.$$

Find the coefficient of (i) skewness (ii) kurtosis. 3

(b) Obtain a recurrence relation for the moments of a normal distribution. 3

7. (a) The Joint probability distribution of random variables X and Y is given by

$P(X = 0, Y = 0) = P(X = 0, Y = 1) = P(X = 0, Y = -1)$ and

$P(X = 0, Y = 0) + P(X = 0, Y = 1) + P(X = 1, Y = -1) = 1.$

Find :

(i) Marginal distributions of X and Y

(ii) Conditional probability distribution of X given $Y = 0$, where X and Y take values –1, 0 and 1.

3

(b) Prove that the coefficient of correlation is independent of change of scale and origin. 3

8. (a) Let X and Y have joint probability density function

$$f(x, y) = \begin{cases} x+y & , \text{ if } 0<x<1, 0<y<1 \\ 0 & , \text{ elsewhere} \end{cases}$$

Find the conditional mean and variance of Y given $X = x, 0 < x < 1$. 3

(b) Let X and Y have a bivariate normal distribution with mean μ_1 and μ_2, variances σ_1^2 and σ_2^2 and correlation coefficient r. Then prove that X and Y are independent iff $r = 0$. 3