(i) Printed Pages: 3 | Roll No. ....

(ii) Questions : 8 | Sub. Code : 0 5 4 1

Exam. Code : 0 0 0 6

B.A./B.Sc. (General) 6th Semester

(2053)

MATHEMATICS

Paper—I : Analysis—II

Time Allowed : Three Hours] [Maximum Marks : 30

Note :—Attempt **five** questions in all, selecting at least **two** questions from each Unit.

UNIT–I

1. (a) Let $A = \{(x, y) | 3 \le x \le 7, 1 \le y \le 5\}$ and $f : A \to \mathbb{R}$ defined as $f(x, y) = \begin{cases} 5 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational} \end{cases}$. Use definition of double integral to show that f(x, y) is not integrable over A.

   (b) Evaluate $\iint_A x \, dxdy$; where

   $$A = \{(x, y) | b^2x^2 + a^2y^2 = a^2b^2; x \ge 0\}.$$ 3+3=6

2. (a) Find the area bounded by $y = x^2$ and $y = x + 2$ using double integration.

   (b) Find the volume of solid;

   $$b^2c^2x^2 + a^2c^2y^2 + a^2b^2z^2 = a^2b^2c^2$$

   using Triple integral. 3+3=6

3. (a) Evaluate $\int_0^1 \int_{y^2}^{y} \frac{y\,dxdy}{(1-x)\sqrt{x-y^2}}$.

(b) Find the work done in moving a particle once around a circle C in XY plane; if circle has centre at the origin and radius Z. The force field is given by :

$\vec{F} = (2x - y + 2z)\hat{i} + (x + y - z)\hat{j} + (3x - 2y - 5z)\hat{k}$.

3+3=6

4. (a) State and prove Green's Theorem in Plane.

(b) Verify Gauss' divergence theorem for $\bar{A} = x\hat{i} + y\hat{j} + z\hat{k}$ over the region bounded by $x^2 + y^2 + z^2 = 16$.

3+3=6

UNIT–II

5. (a) Show that sequence $\{f_n(x)\}$ where

$$f_n(x) = \frac{nx}{1 + n^2x^2},\ x \in \mathbb{R}$$

is not uniformly convergent in any interval that contains zero.

(b) State and prove Weierstrass's M-test for uniform convergence of series. 3+3=6

6. (a) Show that sum function of the series

$$\sum_{n=1}^{\infty}\left[\frac{n^2x}{1 + n^3x^2} - \frac{(n-1)^2x}{1 + (n-1)^3x^2}\right]$$

is continuous $\forall x$ although it does not converge uniformly on [0, 1].

(b) Show that :

$$\int_0^1 \left(\sum_{n=1}^{\infty} \frac{x^n}{n^2} \right) dx = \sum_{n=1}^{\infty} \frac{1}{n^2(n+1)}$$ 3+3=6

7. (a) Find the radius of convergence and interval of convergence of power series $\sum_{n=0}^{\infty} \frac{(x-2)^{4n}}{4^n}$.

(b) Show that :

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots \forall x \in (-1, 1].$$

Hence deduce that log 2 = 1 – 1/2 + 1/3 – 1/4 + ...

3+3=6

8. (a) Obtain the Fourier series in $(-\pi, \pi)$ for a function

$$f(x) = \begin{cases} x & ; -\pi < x \le 0 \\ 2x & ; 0 < x < \pi \end{cases}$$

(b) Express 'sin x' as a cosine series in $(0, \pi)$. 3+3=6