(i) Printed Pages: 3 Roll No. ....

(ii) Questions : 8

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Exam. Code : | 0 | 0 | 0 | 6 |

B.A./B.Sc. (General) 6th Semester

(2053)

MATHEMATICS

Paper-II : Linear Algebra

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** questions from each unit.

UNIT—I

1. (a) Prove that the union of two subspaces is a subspace iff one of them is a subset of the other.

   (b) Examine whether $f(x) = 6x^2 - 4x + 1$ belongs to the linear span of $f(x) = x^2 - x + 1$ and $g(x) = -3x^2 + x + 2$?

   3,3

2. (a) Prove that there exists a basis for each finitely generated vector space.

   (b) Extend $\{(-1, 2, 5)\}$ to two different bases of $R^3|R$).

   3,3

3. (a) Describe explicitly the linear transformation $T : R^2 \to R^3$ such that $T(2, 3) = (4, 5)$ and $T(1, 0) = (0, 0)$.

(b) Let $L : R^4 \to R^3$ be defined by

$L(x, y, z, w) = (x + y, y - z, z - w)$.

Verify Rank Nullity theorem for L. 3,3

4. (a) State and prove first isomorphism theorem of linear transformation.

(b) If T is a linear operator on V such that $T^2 - T + I = 0$. Prove T is invertible. 3,3

UNIT—II

5. (a) If a matrix of a linear operator T on R^3 relative to usual basis is $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}$, then find matrix of T relative to basis $B_1 = \{(0, 1, -1), (-1, 1, 0), (1, -1, 1)\}$.

(b) Let $S = \{(1, 2), (0, 1)\}$ and $T = \{(1, 1), (2, 3)\}$ be bases for R^2. What is transition matrix from basis T to basis S ? 3,3

6. (a) Find all the eigen values and basis for each eigen space of linear operator $T : R^3 \rightarrow R^3$ defined by

$$T(x, y, z) = (3x + y + 4z, 2y + 6z, 5z).$$

(b) Prove that any two characteristic vectors corresponding to two distinct characteristic roots of a hermitian matrix are orthogonal. 3,3

7. (a) For each of the following linear operators, find the characteristic polynomials and verify Cayley-Hamilton theorem $T : R^2 \rightarrow R^2$ defined by $T(x, y) = (x + y, x - y)$.

(b) Find a polynomial whose one root is $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. 3,3

8. (a) Show that the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$ is derogatory.

(b) Let $T : R^3 \rightarrow R^2$ be the linear transformation defined by $T(x, y, z) = (2x + y - z, 3x - 2y + 4z)$. Find the matrix of T relative to ordered basis.

$B_1 = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ and

$B_2 = \{(1, 3), (1, 4)\}$ of R^3 and R^2 respectively. 3,3