(i) Printed Pages : 3 Roll No. ....

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B.A./B.Sc. (General) 4th Semester
(2053)
MATHEMATICS
Paper : I Advanced Calculus–II

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :—Attempt **FIVE** questions in all, selecting at least **TWO** questions from each unit. All questions carry equal marks.

UNIT—I

1. (a) By definition, show that $\lim\limits_{n\to\infty}\dfrac{3n}{n+5\sqrt{n}}=3$.

   (b) Let $\{a_n\}$ be a convergent sequence and $\{b_n\}$ be a sequence diverging to ∞. Show that the sequence $\left\{\dfrac{a_n}{b_n}\right\}$ is a null sequence.

2. (a) Let $\{a_n\}$ be a sequence of positive numbers such that $a_{n+1}=\dfrac{a_n^2+\alpha^2}{2a_n}\ \forall\, n\in N$, a > 0. Show that $\lim\limits_{n\to\infty}a_n=\alpha$.

   (b) Show that the sequence $\{S_n\}$, where

   $$S_n = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+....+\frac{1}{(n-1)!}$$ is convergent.

3. (a) Prove that $\{a_n\}$, where $a_n = \left[\frac{(3n)!}{(n!)^3}\right]^{\frac{1}{n}}$ is convergent.

(b) State the prove Cauchy's second theorem on limits.

4. (a) Let f be a function defined on (0, 1) by

$$f(x) = \begin{cases} 0, & \text{if } x \text{ is an irrational number} \\ \frac{1}{q}, & \text{if } x = \frac{p}{q},\ p, q \in N, (p, q) = 1. \end{cases}$$

Show that f is continuous at each irrational point and discontinuous at each rational point.

(b) Show that $f(x) = x^2$ is uniformly continuous on (–1, 1].

UNIT—II

5. (a) Discuss the convergence of $\sum_{n=1}^{\infty} u_n$, where a > 1 and

$$u_n = a^{\frac{1}{n}} - 1 - \frac{1}{n}\log a.$$

(b) Prove that $\lim_{n\to\infty}\left(1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n} - \log n\right)$ exists and lies between 0 and 1.

6. (a) Discuss the convergence and divergence of the series

$$\sum_{n=1}^{\infty}\sqrt{\frac{n+1}{n^2+1}}\,x^n.$$

(b) Examine the convergence of the series

$$\sum_{n=1}^{\infty}(-1)^n\left(\frac{1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}}{n}\right).$$

7. (a) Show that the following series is conditionally convergent $\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{1}{3n-1}\right)$.

(b) Examine the convergence or divergence of the series

$$1+\frac{2^2}{3^2}+\frac{2^2.4^2}{3^2.5^2}+\frac{2^2.4^2.6^2}{3^2.5^2.7^2}+\ldots\ldots.$$

8. (a) Show that the series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}x^{2n-1}}{(2n-1)!}$ is convergent $\forall x \in R$.

(b) What rearrangement of the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\ldots..$ will reduce its sum to 0 ?