(i) Printed Pages : 3 Roll No. .....

(ii) Questions : 8 Sub. Code : 0145

Exam. Code : 0002

B.A./B.Sc. (General) 2nd Semester

(2053)

MATHEMATICS

Paper–I : Solid Geometry

Time Allowed : Three Hours] [Maximum Marks : 30

Note :—Attempt FIVE questions in all, selecting at least TWO questions from each unit.

UNIT—I

1. (a) Shift the origin to a suitable point so that the equation $x^2 + y^2 + z^2 - 4x - 8y + 6z - 4 = 0$ is transformed into an equation in which first degree terms are absent.

(b) Transform the equation $13x^2 + 13y^2 + 10z^2 + 8xy - 4yz - 4zx - 144 = 0$, when the axes are rotated to the axes having direction cosines $<-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}>$, $<\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}>$ and $<\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}>$. 2×3=6

2. (a) Show that the two circles

$x^2 + y^2 + z^2 - y + 2z = 0$, $x - y + z - 2 = 0$;

$x^2 + y^2 + z^2 + x - 3y + z - 5 = 0$,

$2x - y + 4z - 1 = 0$ lie on the same sphere and find its equation.

(b) Find the equation of the sphere which touches the plane $3x + 2y - z = 6$ at the point P(1, 2, 1) and cuts orthogonally the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$.

2×3=6

3. (a) Find the equation of the cylinder whose generators are parallel to $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and guiding curve is $x^2 + y^2 = 16$, $z = 0$.

(b) Find the equation of the right circular cylinder of radius 2, whose axis is the line $\frac{x-1}{2} = \frac{y}{1} = \frac{z-3}{2}$.

2×3=6

4. (a) Find the limiting points of the co-axial system determined by two spheres whose equations are

$x^2 + y^2 + z^2 - 8x + 2y - 2z + 32 = 0$ and

$x^2 + y^2 + z^2 - 7x + z + 23 = 0$.

(b) Find the enveloping cylinder of the sphere $x^2 + y^2 + z^2 = 25$, whose generators are parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$.

2×3=6

UNIT—II

5. (a) Find the equation of right circular cone which passes through (1, 1, 1), whose vertex is (1, 0, 1) and axis of cone makes equal angles with co-ordinate axes.

   (b) Find the equation of the cone which passes through the co-ordinate axes and the lines $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$, $\frac{x}{3} = \frac{y}{-1} = \frac{z}{1}$. $2\times3=6$

6. (a) Find the equation of the cone whose vertex is at the point (0, 0, 5) and whose guiding curve is $z = 3$, $x^2 + 2xy + 3y^2 = 1$.

   (b) Prove that the equation

   $$x^2 - 2y^2 + 3z^2 - 4xy + 5yz - 6zx + 8x - 19y - 2z - 20 = 0$$

   represents a cone. Find its vertex. $2\times3=6$

7. (a) Identify the following surface :

   $$2x^2 + 2y^2 + 3z^2 - 4x + 8y - 12z + 16 = 0.$$

   (b) Find the equation of the surface generated by the revolution of the circle $x^2 + y^2 - 2ay + a^2 - r^2 = 0$; $z = 0$ about the x-axis ($a > r$). $2\times3=6$

8. Prove that $5x^2 - 16y^2 + 5z^2 + 8yz - 14zx + 8xy + 4x + 20y + 4z - 24 = 0$ represents hyperbolic paraboloid. 6