9/8/2023 (Morning)

(i) Printed Pages: 3 Roll No. .....

(ii) Questions : 8 Sub. Code : 0147

Exam. Code : 0002

B.A./B.Sc. (General) 2nd Semester

(2053)

MATHEMATICS

Paper—III : Theory of Equations

Time Allowed : Three Hours] [Maximum Marks : 30

Note :—Attempt FIVE questions in all, selecting at least TWO questions from each Unit. All questions carry equal marks.

UNIT—I

1. (a) Solve the equation $x^4 - 4x^2 + 8x - 4 = 0$ when one of its roots is $1 + i$. 3

(b) Find k so that the equation $2x^4 - 3x^2 - 2x + k = 0$ may have a double root and solve the equation. 3

2. (a) Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ $(a \neq 0, d \neq 0)$ may be in G.P. Hence solve the equation $27x^3 + 42x^2 - 28x - 8 = 0$. 3

(b) Two roots of the equation $x^4 + 6x^3 - 16x^2 + 24x - 80 = 0$ are purely imaginary. Solve the equation completely. 3

3. (a) Remove the second term form the equation

$$4x^4 + 32x^3 + 83x^2 + 76x + 21 = 0$$

and hence solve it completely. 3

(b) If α, β, γ are roots of $4x^2 - 12x^2 + 5x + 6 = 0$, then find the equation whose roots are $\alpha - \frac{1}{2}, \beta - \frac{1}{2}, \gamma - \frac{1}{2}$. Solve the same. 3

4. (a) State and prove Descarte's rule of signs. 3

(b) Find the equation whose roots are squared differences of the roots of the cubic $x^3 - 6x + 4\sqrt{2} = 0$. Hence prove that the given equation has a multiple root. 3

UNIT—II

5. (a) Apply Newton's Method of divisors to solve the equation $4x^5 + 4x^4 - 27x^3 - 8x^2 + 51x - 18 = 0$. 3

(b) Use Cardan's method to solve the cubic :

$$2x^3 + 3x^2 + 3x + 1 = 0$$ 3

6. (a) Use Descarte's method to solve :

$$x^4 - 15x^2 + 20x - 6 = 0$$ 3

(b) Prove that the discriminant of the cubic equation $a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$ is $\frac{-27}{a_0^2}(G^2 + 4H^3)$. 3

7. (a) Use Ferrari's method to solve :

$$x^4 + 8x^3 + 9x^2 - 8x - 10 = 0$$ 3

(b) For an equation $3x^4 - 8x^3 - 30x^2 + 72x + 5 = 0$, find an interval in which real root lies. 3

8. (a) Find by Newton's method of approximation the root near to 2 of $x^4 - 12x + 7 = 0$ correct to three decimal places. 3

(b) Discuss the nature of roots of the equation :

$$2x^3 - 7x^2 + 8x - 3 = 0$$ 3