(i) Printed Pages : 4 Roll No. ....

(ii) Questions : 9

Sub. Code : | 3 | 7 | 0 | 2 |

Exam. Code : | 0 | 4 | 7 | 2 |

M.Sc. Physics 1st Semester

(2123)

MATHEMATICAL PHYSICS–I

Paper : PHY–8011

Time Allowed : Three Hours] **[Maximum Marks : 60**

Note:— Attempt **five** questions in all, including Q.No. 9 of Unit–V is compulsory and selecting **one** question from each Unit I–IV.

UNIT—I

1\. (a) State and prove Cauchy Riemann conditions in polar form. 6

(b) Using calculus of Residues, Evaluate the integral

$$\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta}\, d\theta .$$ 6

2\. (a) Evaluate $\int_{-\infty}^{+\infty} \frac{x^2\,dx}{(x^2+a^2)(x^2-b^2)}$, a, b > 0. 6

(b) What are Dispersion relations ? Obtain them for a complex function f(x), where x is real variable. 6

UNIT—II

3. (a) Give various Definitions of Gamma function and show that the Gamma function of Weistrass's form leads to an important identity :

$$\Gamma(z)\,\Gamma(1-z) = \frac{\pi}{\sin \pi z}.$$ 6

(b) Verify that $\Gamma(z) = a^z \int_0^{\infty} e^{-at}\, t^{z-1}\, dt,\ z, a > 0.$ 3

(c) Evaluate $\int_0^{\pi/2} (\sin\theta)^p (\cos\theta)^q$. 3

4. (a) Show that $\delta[f(x)] = \left(\frac{df(x)}{dx}\right)^{-1} \delta(x - x_0)$

where x_0 is so chosen that $f(x_0) = 0$. 6

(b) Show that $\int_{-\infty}^{+\infty} \delta'(x)\, f(x)\, dx = f'(0)$. 3

(c) Obtain the relation between Beta and Gamma function. 3

UNIT—III

5. (a) Obtain the series solution of linear harmonic oscillator $\frac{d^2y}{dx^2} + w_o^2\, y = 0$ using Frobenius method. 6

(b) Write down the Laplace's equation in two dimensions in polar coordinates (r, θ) and solve it using method of separation of variable. 6

6. (a) Find the inverse of the matrix $A = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 2 & 3 \\ 3 & -3 & 1 \end{bmatrix}$. 6

(b) Develop the method of finding second linear solution of a homogeneous second order differential equation if one of its solution is known. 6

UNIT—IV

7. (a) State and prove the integral representation of the Bessel function. 6

(b) Obtain the Rodrigue formula for Legendre polynomials and hence find the value of $P_2(x)$ and $P_3(x)$. 6

8. (a) State and prove orthogonality condition for the Hermite function. 6

(b) Show that :

(i) $(n + 1) L_{n+1}(x) = (2n + 1 - x) L_n(x) - n L_{n-1}(x)$

(ii) $P'_{n+1}(x) - P'_{n-1}(x) = (2n + 1) P_n(x)$. 6

UNIT—V

9. (a) What is Harmonic function ? Is the function u(x, y) = sin cos hy ?

   (b) Find the analytic function w(z), if $u(x, y) = e^{-y} \sin x$.

   (c) Discuss various types of singular points in a given differential equation.

   (d) What is Wronskian ? What is its use in differential equation ?

   (e) Find the Laurent expansion of $f(z)=\frac{1}{z(z-1)}$ about $z=-1$.

   (f) Is Beta function is symmetric w.r.t. its argument ? 6×2