(i) Printed Pages : 3 Roll No. .....

(ii) Questions : 8 Sub. Code : 0 0 4 3

Exam. Code : 0 0 0 1

B.A./B.Sc. (General) 1st Semester

(2123)

MATHEMATICS

Paper—I : Plane Geometry

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt **five** questions in all, selecting at least **two** questions from each section. All questions carry equal marks.

UNIT—I

1. (a) Show that by a suitable translation of axes, the first degree terms can be removed from the equation $x^2 + 3xy - y^2 + 3x - 7y + 11 = 0$.

   (b) Find the joint equation of two straight lines passing through (1, 2) and perpendicular to lines $3x^2 - 8xy + 5y^2 = 0$.

2. (a) Find equation of the bisectors of the angle between the lines joining origin to the points of intersection of curve $x^2 + xy + y^2 + x + 3x + 1 = 0$ and the straight line $x + y + 2 = 0$.

   (b) Find the equation of a circle described on the common chord of circle $x^2 + y^2 - 6x - 4y - 12 = 0$ and $x^2 + y^2 - 2x - 6y - 15 = 0$ as diameter.

3. (a) Find the equation of a circle which touches both the coordinate axes and the line $3x - 4y + 8 = 0$ and lies in the third quadrant.

(b) If pole of a line w.r.t. Circle $x^2 + y^2 = a^2$ lies on Circle $x^2 + y^2 = 9a^2$, then show that polar is a tangent to the Circle $9(x^2 + y^2) = a^2$.

4. (a) Find the equation of the circle which belongs to the co-axial system of which the limiting points are $(1, -1)$ and $(2, 0)$ and which passes through origin.

(b) Find the radical axis and the limiting points of the system of co-axial circles

$3(x^2 + y^2) - 16x - 14y + 39 + \lambda(x^2 + y^2 - 5x - 5y + 13) = 0$

UNIT—II

5. (a) Find the equation of the common tangent to the parabolas $y^2 = 4ax$ and $x^2 = 4by$.

(b) Prove that the length of chord of contact of tangents drawn from an exterior point (α, β) to the parabola $y^2 = 4ax$ is

$$\frac{1}{a}\sqrt{\beta^2 + 4a^2}\,\sqrt{\beta^2 - 4a\alpha}\,.$$

6. (a) Find the locus of the middle points of chords of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, which are of constant length 2d.

(b) Show that the minimum angle between a pair of conjugate diameter of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\tan^{-1}\left(\frac{2ab}{a^2 - b^2}\right)$.

7. (a) Find the eccentricity of the hyperbola of which $2x - 3y = 0$ and $x = 2y$ is a pair of conjugate diameters.

(b) Find the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 5x + 11y - 8 = 0$. Also find the equation of its conjugate hyperbola.

8. (a) Prove that the locus of the middle points of normal chords of the rectangular hyperbola $x^2 - y^2 = a^2$ is $(y^2 - x^2)^3 = 4a^2 x^2 y^2$.

(b) Identify the curve $4x^2 + y^2 - 8x + 2y + 1 = 0$. Also, find foci, eccentricity and latus rectum.